More facts about continuous random variables and their expectations: We know $E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$. Using $g(X) = aX + b$ for constants a, b, we get

$$E(aX+b) = \int_{-\infty}^{\infty} (ax+b) f_X(x)\,dx = a \int_{-\infty}^{\infty} x\,dx + b \int_{-\infty}^{\infty} f_X(x)\,dx = aE(X) + b.$$

So in summary $E(aX+b) = aE(X) + b$, just as it was for discrete random variables.

What if we want to take the expected value of a function of two random variables?

$$E(g(X,Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y)\,dy\,dx.$$

In particular, if we use $g(X,Y) = X + Y$, we get

$$E(X+Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x+y) f_{X,Y}(x,y)\,dy\,dx$$

We can split this up into two parts, namely, $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f_{X,Y}(x,y) dy dx = \int_{-\infty}^{\infty} x \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy dx = \int_{-\infty}^{\infty} x f_X(x)\,dx = E(X)$. [Note: remember that $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$, i.e., we just integrate y out of the picture.] Similarly, the second term in $E(X+Y)$ is just $E(Y)$ (please check!). So altogether we get

$$E(X+Y) = E(X) + E(Y),$$

just as it was true also for discrete random variables. As with discrete random variables, we do not even need to know that X and Y are independent for this to hold. I.e., $E(X+Y) = E(X) + E(Y)$ for all continuous random variables.