

Another nice fact, same as for discrete random variables: If X and Y are independent (we should only be using this rule if we are sure we have independence), then

$$E(XY) = E(X)E(Y).$$

More generally,

$$E(g(X)h(Y)) = E(g(X))E(h(Y)),$$

for any functions g and h . It's enough to show that this second rule works, because if we use $g(X) = X$ and $h(Y) = Y$, this shows automatically, afterwards, that the first rule works too. So we show how to verify the second rule, which is the more general one.

$$\begin{aligned} E(g(X)h(Y)) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x)h(y)f_{X,Y}(x,y) dy dx \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x)h(y)f_X(x)f_Y(y) dy dx && \text{since } X, Y \text{ independent} \\ &= \int_{-\infty}^{\infty} g(x)f_X(x) dx \int_{-\infty}^{\infty} h(y)f_Y(y) dy \\ &= E(g(X))E(h(Y)) \end{aligned}$$