Exponential random variables. We say that a random variable X is an exponential random variable if X has probability density function $f_X(x) = \lambda e^{-\lambda x}$ for $x > 0$, and $f_X(x) = 0$ otherwise. We have seen some examples with this kind of density function before. Many times, when we see an exponential random variable, the random variable represents some kind of waiting time for some random event to occur. Key idea: waiting times! For instance, the time until the doorbell rings, or the next email arrives, or the telephone rings, or the next blue car passes on the street, etc., etc.

First find the CDF of an exponential random variable X. Easiest to find $P(X > a$, for $a > 0$. To do this,

$$P(X > a) = \int_a^\infty \lambda e^{-\lambda x}\, dx = \frac{\lambda e^{-\lambda x}}{-\lambda}|_{x=a}^{\infty} = e^{-\lambda a}.$$

Therefore, the CDF of X is equal to

$$F_X(a) = P(X \leq a) = 1 - P(X > a) = 1 - e^{-\lambda a},$$

for $a > 0$, and $F_X(a) = 0$ otherwise, i.e., for $a \leq 0$.