

Suppose X is an exponential random variable with density

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X) = \int_0^{\infty} x \lambda e^{-\lambda x} dx = \underbrace{(x)(-e^{-\lambda x}) \Big|_{x=0}^{\infty}}_{\text{use int. by parts.}} - \underbrace{\int_0^{\infty} -e^{-\lambda x} dx}_{\text{focus on this part}}$

$u = x$
 $dv = \lambda e^{-\lambda x} dx$ $du = dx$ $v = \frac{de^{-\lambda x}}{-\lambda} = -e^{-\lambda x}$

$= 0$ for $x=0$
 for $x \rightarrow \infty$
 $\rightarrow \frac{\infty}{\infty}$ written as $\frac{-x}{\lambda x}$
 need L'Hospital's rule
 $= \lim_{x \rightarrow \infty} \frac{-1}{(e^{-\lambda x} \lambda)} = 0$

$$E(X) = \int_0^{\infty} e^{-\lambda x} dx = \frac{e^{-\lambda x}}{-\lambda} \Big|_{x=0}^{\infty} = \frac{1}{\lambda} \quad \text{so } \boxed{E(X) = \frac{1}{\lambda}}$$

$E(X^2) = \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx = \underbrace{(x^2)(-e^{-\lambda x}) \Big|_{x=0}^{\infty}}_{= 0 - 0 = 0} - \int_0^{\infty} (2x)(-e^{-\lambda x}) dx$

int. by parts.
 $u = x^2$ $du = 2x dx$
 $dv = \lambda e^{-\lambda x} dx$ $v = \frac{\lambda e^{-\lambda x}}{-\lambda} = -e^{-\lambda x}$

$= \frac{2}{\lambda} \int_0^{\infty} x \lambda e^{-\lambda x} dx$
 (put in λ and divide by λ)
 $= \frac{2}{\lambda} \cdot E(X) = \frac{2}{\lambda} \cdot \frac{1}{\lambda} = \frac{2}{\lambda^2}$

$\text{Var}(X) = E(X^2) - (E(X))^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$ Also standard deviation of X is $\frac{1}{\lambda}$.