

### Gamma random variables

The idea of a Gamma random variable is to sum together a finite (fixed) number of independent exponential random variables.

We have seen a concept like this before, when we studied discrete random variables. Think back to when we learned about Geometric random variables. Remember that if (for example)  $X_1, X_2, X_3, X_4, X_5$  are Geometric random variables that are independent, then  $X = X_1 + X_2 + X_3 + X_4 + X_5$ , then  $X$  is a Negative Binomial random variable. Remember that  $X$  was the number of trials until the 5th success occurs. In general, remember that a Negative Binomial random variable is the number of trials until the  $r$ th success, where  $r$  is some fixed number. This gives some context to the motivation for Gamma random variables.

Following this same kind of example, let's suppose that  $X_1, X_2, X_3, X_4, X_5$  are exponential random variables that are independent. Then their sum  $X = X_1 + X_2 + X_3 + X_4 + X_5$  is called a Gamma random variable with parameter  $r = 5$ . Just like with Geometric versus Negative Binomial random variables, we need a second parameter too, i.e., we need to know the value of the parameter  $\lambda$  that comes from the Exponential random variables, and moreover, this parameter  $\lambda$  has to be the same for all of the independent Exponential random variable that we sum to build our Gamma random variable.

So if  $X_1, \dots, X_5$  are independent Exponential random variables all with a common parameter  $\lambda$  (say, for instance,  $\lambda = 3$ ), then  $X = X_1 + \dots + X_5$  is a Gamma random variable with parameters  $\lambda = 3$  and  $r = 5$ . The  $r$  tells the number of random variables we are summing up.

Think of Gamma random variables as a sum of waiting times. Remember that an Exponential random variable is often a waiting time for something to occur. For instance, suppose that each  $X_j$  is the waiting time until the next email arrives in our inbox. Then the  $X$  we defined above, i.e., the sum of 5 independent  $X_j$ 's would be the time until the 5th email arrives.

This is the general idea too. A Gamma random variable is often thought of as the time until the  $r$ th occurrence of some kind of repeating event, where the times between consecutive events are independent Exponential random variables.