Example: Suppose \( X \) has density \( f_X(x) = \frac{7^3}{(3-1)!} x^{3-1} e^{-7x} \) for \( x > 0 \)

\[ = 0 \text{ otherwise} \]

and suppose \( Y \) has density \( f_Y(y) = \frac{7^5}{(5-1)!} y^{5-1} e^{-7y} \) for \( y > 0 \)

\[ = 0 \text{ otherwise} \]

and suppose \( X \) and \( Y \) are independent. Q: What kind of random variable is \( X + Y \)?

Notice that \( X \) has the same distribution as \( X_1 + X_2 + X_3 \)
where the \( X_j \)'s are independent Exponential random variables, each with \( \lambda = 7 \).

Notice that \( Y \) has the same distribution as \( Y_1 + Y_2 + Y_3 + Y_4 + Y_5 \)
where the \( Y_j \)'s are independent Exponential random variables, each with \( \lambda = 7 \).

So \( X + Y \) has the same distribution as the sum of 8 independent Exponential random variables, each with \( \lambda = 7 \).

So \( X + Y \) is a Gamma random variable with \( \lambda = 7, r = 8 \).

More generally, if we sum several independent Gamma random variables with a common parameter \( \lambda \) and possibly different \( r \) values, then the sum is also a Gamma random variable with the same parameter \( \lambda \) and with \( r \) equal to the sum of the \( r \)'s of the Gamma random variables that make up the sum.