Expected value and variance of a Beta random variable

In general:  \( E(X) = \frac{\alpha}{\alpha + \beta} \)  \( Var(X) = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)} \)

Do not need to derive these in the general case.

We focus on specific \( \alpha \) and \( \beta \) values, and when we fix specific \( \alpha \) and \( \beta \) then we can derive the expected value and variance.

Notice:  \( f_X(x) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & \text{for } x \in [0,1] \\ 0 & \text{otherwise} \end{cases} \)

This \( \Gamma \) function is just a factorial of the input minus 1, when the input is an integer, i.e. \( \Gamma(n) = (n-1)! \) for integers \( n \).

\( \Gamma(5) = 4! \), \( \Gamma(8) = 7! \), etc.