

Example Suppose $\alpha = 3, \beta = 8$.

Then X has density $f_x(x) = \begin{cases} \frac{(\alpha+\beta-1)!}{(\alpha-1)!(\beta-1)!} x^{\alpha-1} (1-x)^{\beta-1} & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$$f_x(x) = \frac{10!}{2!7!} x^2 (1-x)^7 \quad \text{for } 0 < x < 1$$

Find the CDF of X .

$$F_x(x) = 0 \quad \text{for } x \leq 0$$

$$F_x(x) = 1 \quad \text{for } x \geq 1$$

for $0 < a < 1$

$$F_x(a) = \int_0^a f_x(x) dx = \int_0^a \underbrace{360 x^2 (1-x)^7}_{\text{could expand, but painful}} dx$$

Hint: If necessary, do u -sub so that x is raised to the larger power and $1-x$ to the smaller power.

$$u = 1-x \quad x = 1-u \quad du = -dx$$

$$\begin{aligned} F_x(a) &= \int_1^{1-a} 360(1-u)^2 u^7 (-1) du = \int_{1-a}^1 360(1-2u+u^2)u^7 du \\ &= \int_{1-a}^1 360(u^7 - 2u^8 + u^9) du \\ &= 360 \left(\frac{u^8}{8} - 2\frac{u^9}{9} + \frac{u^{10}}{10} \right) \Big|_{u=1-a}^1 \\ &= 360 \left[\left(\frac{1}{8} - \frac{2}{9} + \frac{1}{10} \right) - \left(\frac{(1-a)^8}{8} - \frac{2(1-a)^9}{9} + \frac{(1-a)^{10}}{10} \right) \right] \end{aligned}$$