

How do we derive the expected value of a Beta random variable?

Consider the case  $\alpha = 3$ ,  $\beta = 8$ , so  $X$  has density  $f_X(x) = 360x^2(1-x)^7$   
for  $0 < x < 1$

$$E(X) = \int_0^1 (x)(360x^2(1-x)^7) dx$$

$$\begin{aligned} u &= 1-x & du &= -dx \\ x &= 1-u \end{aligned}$$

easier if  $x$  has the larger power because we do not have to expand it, and if  $(1-x)$  has the smaller power, because then, when we expand it, we get fewer terms.

$$= \int_1^0 (1-u)(360)(1-u)^2 u^7 (-1) du$$

$$= 360 \int_0^1 (1-3u+3u^2-u^3) u^7 du$$

$$= 360 \int_0^1 (u^7 - 3u^8 + 3u^9 - u^{10}) du$$

$$= 360 \left( \frac{u^8}{8} - \frac{3}{9} u^9 + \frac{3}{10} u^{10} - \frac{1}{11} u^{11} \right) \Big|_{u=0}^1$$

$$= 360 \left( \frac{1}{8} - \frac{3}{9} + \frac{3}{10} - \frac{1}{11} \right)$$

$$= 360 \left( \frac{1}{1320} \right) = \frac{3}{11} = \frac{\alpha}{\alpha + \beta} \text{ as claimed in the general case.}$$