

Variance of a Beta random variable. Assume $\alpha = 3, \beta = 8$

$$f_x(x) = 360x^2(1-x)^7 \text{ for } 0 < x < 1$$

$$E(X^2) = \int_0^1 (x^2)(360)x^2(1-x)^7 dx \quad \begin{array}{l} u = 1-x \\ x = 1-u \end{array} \quad du = -dx$$

$$= \int_1^0 360(1-u)^4 u^7 (-1) du$$

$$= 360 \int_0^1 (1-4u+6u^2-4u^3+u^4)u^7 du$$

$$= 360 \int_0^1 (u^7 - 4u^8 + 6u^9 - 4u^{10} + u^{11}) du$$

$$= 360 \left(\frac{u^8}{8} - \frac{4}{9}u^9 + \frac{6}{10}u^{10} - \frac{4}{11}u^{11} + \frac{u^{12}}{12} \right) \Big|_{u=0}^1$$

$$= 360 \left(\frac{1}{8} - \frac{4}{9} + \frac{6}{10} - \frac{4}{11} + \frac{1}{12} \right) = 360 \left(\frac{1}{3960} \right) = \frac{1}{11}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = \frac{1}{11} - \left(\frac{3}{11} \right)^2 = \frac{2}{121} \text{ as claimed earlier.}$$