Expected value and variance of standard normal random variable.

If X is a standard Normal random variable, usually use Z instead of X for its name, so we instantly recognize it. We will be transforming normal random variables a lot. So it is helpful to have an instantly recognizable letter for a standard Normal random variable.

Let's show that such a Z has mean 0.

$$E(Z) = \int_{-\infty}^{\infty} (z) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0 - 0 = 0$$

Well made for u-sub:

$u = z^2 \quad du = 2z\, dz$

$$\int \underbrace{z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz}_{= \frac{1}{2} du} = \int \frac{1}{2}\frac{1}{\sqrt{2\pi}} e^{-u/2} du = \frac{1}{2}\frac{1}{\sqrt{2\pi}} \frac{e^{-u/2}}{-1/2} = \frac{-1}{\sqrt{2\pi}} e^{-u/2}$$

$$= \frac{-e^{-z^2/2}}{\sqrt{2\pi}}$$

$$Var(Z) = E(Z^2) - \underbrace{(E(Z))^2}_{=0}$$

$$= \int_{-\infty}^{\infty} z^2 \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \underbrace{(z)\left(-\frac{e^{-z^2/2}}{\sqrt{2\pi}}\right)\Bigg|_{z=-\infty}^{\infty}}_{=0-0=0} - \int -\frac{e^{-z^2/2}}{\sqrt{2\pi}} dz = 1$$

by L'Hospitals Rule

can be +

use int. by parts.

$u = z \qquad du = dz$

$dv = z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \qquad v = -\frac{e^{-z^2/2}}{\sqrt{2\pi}}$

Used the fact that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 1$ i.e. we used the fact that this is an honest-to-goodness density function, even though we did not show it.

Know now $E(Z) = 0$, $Var(Z) = 1$. ✓