Probability a normal random variable is within 1, 2, or 3 standard deviations of its mean:

$$P(\mu_X - \sigma_X < X < \mu_X + \sigma_X) = P\left(\frac{\mu_X - \sigma_X - \mu_X}{\sigma_X} < \frac{X - \mu_X}{\sigma_X} < \frac{\mu_X + \sigma_X - \mu_X}{\sigma_X}\right)$$

$$= P(-1 < Z < 1)$$

$$= P(Z < 1) - P(Z < -1)$$

$$= P(Z < 1) - (1 - P(Z < 1))$$

$$= 0.8413 - (1 - 0.8413)$$

$$= 0.6826$$

i.e. a Normal random variable is within 1 standard deviation of its mean with probability 68%.

$$\text{Similarly, } P(\mu_X - 2\sigma_X < X < \mu_X + 2\sigma_X) = P(Z < 2) - (1 - P(Z < 2))$$

$$= 0.9772 - (1 - 0.9772)$$

$$= 0.9544$$

i.e. X is within two standard deviations of its mean with prob 95%.

$$\text{Similarly } P(\mu_X - 3\sigma_X < X < \mu_X + 3\sigma_X) = P(Z < 3) - (1 - P(Z < 3))$$

$$= 0.9987 - (1 - 0.9987)$$

$$= 0.9974$$

i.e. X is within 3 std. dev. of its mean with prob 99.74%.