

Example Suppose X_1, X_2, \dots, X_{75} each have mean 1.5 and variance 2. Find the probability that the sum of the X_j 's is less than 95.

$$\begin{aligned} P(X_1 + X_2 + \dots + X_{75} < 95) &= P\left(\frac{X_1 + X_2 + \dots + X_{75} - (75)(1.5)}{\sqrt{75(2)}} < \frac{95 - (75)(1.5)}{\sqrt{75(2)}}\right) \\ &= P(Z < -1.43) \quad \text{where } Z \text{ is a standard} \\ &= P(Z > 1.43) \quad \text{Normal random variable} \\ &\quad \downarrow \text{by symmetry} \\ &= 1 - P(Z \leq 1.43) \\ &= 1 - F_Z(1.43) = 1 - 0.9236 = 0.0764 \end{aligned}$$