

Example Suppose X_1, X_2, \dots, X_{150} each have expected value 3.5 and variance 1.2. Find a threshold "a" so that

$$P(X_1 + X_2 + \dots + X_{150} \leq a) = .90$$

i.e. want a value "a" so that we are 90% sure the sum of the X_j 's is less than or equal to "a".

$$\begin{aligned} .90 &= P(X_1 + \dots + X_{150} \leq a) = P\left(\frac{X_1 + \dots + X_{150} - (150)(3.5)}{\sqrt{150(1.2)}} \leq \frac{a - (150)(3.5)}{\sqrt{150(1.2)}}\right) \\ &= P\left(Z \leq \frac{a - (150)(3.5)}{\sqrt{150(1.2)}}\right) \\ &= F_Z\left(\frac{a - (150)(3.5)}{\sqrt{150(1.2)}}\right) \end{aligned}$$

↑ know this input to the CDF should be about 1.28
 $F_Z(1.28) = 0.8997$

Need "a" to satisfy $\frac{a - (150)(3.5)}{\sqrt{150(1.2)}} = 1.28$

$$a = 1.28 \sqrt{150(1.2)} + (150)(3.5) = 542.17$$

So we are 90% sure that $X_1 + \dots + X_{150}$ is less than 542.17.