

## Difference of two Normal random variables.

If  $X, Y$  independent Normal random variables, then

$X - Y$  is a Normal random variable too (because  $-Y$  is Normal)  
with mean  $\mu_x - \mu_y$  and variance  $\sigma_x^2 + \sigma_y^2$ .

$$P(X - Y \leq 5) = P\left(\frac{X - Y - (\mu_x - \mu_y)}{\sqrt{\sigma_x^2 + \sigma_y^2}} \leq \frac{5 - (\mu_x - \mu_y)}{\sqrt{\sigma_x^2 + \sigma_y^2}}\right)$$

E.g. If  $\mu_x = 10$      $\mu_y = 2$   
 $\sigma_x^2 = 7$          $\sigma_y^2 = 12$

$$\begin{aligned} &= P\left(Z \leq \frac{5 - (10 - 2)}{\sqrt{7 + 12}}\right) \\ &= P(Z \leq -0.69) \\ &= P(Z \geq 0.69) \\ &= 1 - P(Z \leq 0.69) \\ &= 1 - F_Z(0.69) \\ &= 1 - 0.7549 = 0.2451 \end{aligned}$$