Example: Suppose $Y$ is a Binomial random variable with $n=5000$ and $p = \frac{1}{10}$. Find $P(Y \leq 520)$. We can use CLT to find a good approximation to this probability. The exact value, by the way, is

$$\sum_{y=0}^{520} \binom{5000}{y} \left(\frac{1}{10}\right)^{y}\left(\frac{9}{10}\right)^{5000-y}$$

521 terms here! Difficult to calculate. CLT approximation is more feasible to actually carry out.

CLT is applicable because $Y$ has the same distribution as

$$X_1 + X_2 + \ldots + X_{5000},$$

where the $X_j$'s are independent Bernoulli random variables, each with $p = \frac{1}{10}$. $E(X_j) = \frac{1}{10}$, $\text{Var}(X_j) = \left(\frac{1}{10}\right)\left(\frac{9}{10}\right)$.

**Continuity Correction**

$$P(Y \leq 520)$$

$\uparrow$

$Y$ is integer valued

We split the difference and use $P(Y \leq 520.5)$

$$= P\left(\frac{Y - (5000)(\frac{1}{10})}{\sqrt{5000\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)}} \leq \frac{520.5 - (5000)(\frac{1}{10})}{\sqrt{5000\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)}}\right)$$

$$\approx P(Z \leq 0.97) = 0.8340.$$ 

We are using a continuous random variable to approximate the behavior of a discrete random variable.

We chose 520.5 because any cutoff from 520.0 to 520.999... would be OK and we want to avoid error in rounding as much as possible.