

## Why we need the concept of covariance

Want to do is calculate the variance of a sum of random variables, where these random variables may or may not be independent.

$$\begin{aligned}\text{Var}\left(\sum_{i=1}^n X_i\right) &= E\left(\left(\sum_{i=1}^n X_i\right)^2\right) - \left(E\left(\sum_{i=1}^n X_i\right)\right)^2 \\ &= E\left(\left(\sum_{i=1}^n X_i\right)\left(\sum_{j=1}^n X_j\right)\right) - E\left(\sum_{i=1}^n X_i\right)E\left(\sum_{j=1}^n X_j\right) \\ &= \sum_{i=1}^n \sum_{j=1}^n E(X_i X_j) - \sum_{i=1}^n \sum_{j=1}^n E(X_i)E(X_j) \\ &= \sum_{i=1}^n \sum_{j=1}^n \underbrace{\left(E(X_i X_j) - E(X_i)E(X_j)\right)}\end{aligned}$$

equivalent to the covariance of  $X_i$  and  $X_j$   
So the covariance of two random variables plays a really important role when taking a sum of random variables and finding the variance of such a sum.