Comparison of the definition and use of variances vs covariances

Variance of X is defined as

$$Var(X) = E((X-E(X))^2)$$
$$= E((X-E(X))(X-E(X)))$$

Covariance of X and Y is defined as

$$Cov(X,Y) = E((X-E(X))(Y-E(Y)))$$

Notice if we use $Y=X$ then

$Var(X) = Cov(X,X)$.

Also, we do not need X, Y to be independent, to apply the concept of $Cov(X,Y)$.

Usage of $Var(X)$ is often

$$Var(X) = E((X-E(X))(X-E(X)))$$
$$= E(X^2 - XE(X) - E(X)\cdot X + (E(X))^2)$$
$$= E(X^2) - (E(X))^2 - (E(X))^2 + (E(X))^2$$
$$= E(X^2) - (E(X))^2$$

Similarly, for covariance, often use

$$Cov(X,Y) = E((X-E(X))(Y-E(Y)))$$
$$= E(XY - XE(Y) - YE(X) + E(X)E(Y))$$
$$= E(XY) - E(X)E(Y) - E(X)E(Y) + E(X)E(Y)$$
$$= E(XY) - E(X)E(Y)$$

again notice $Cov(X,X) = Var(X)$

Also this is the form in our earlier discussion about variance of a sum of random variables.