Comparison of the definition and use of variances vs covariances

**Variance of X is defined as**

\[ \text{Var}(X) = E((X-E(X))^2) \]

\[ = E((X-E(X))(X-E(X))) \]

**Covariance of X and Y**

is defined as

\[ \text{Cov}(X, Y) = E((X-E(X))(Y-E(Y))) \]

Notice if we use \( Y = X \) then

\[ \text{Var}(X) = \text{Cov}(X, X). \]

Also, we do not need \( X, Y \) to be independent, to apply the concept of \( \text{Cov}(X, Y) \).

\[ \begin{align*}
\text{Var}(X) &= E(X^2) - (E(X))^2, \\
\text{Cov}(X, Y) &= E(XY) - E(X)E(Y) \\
\end{align*} \]

Similarly, for covariance often use

\[ \begin{align*}
\text{Cov}(X, Y) &= E((X-E(X))(Y-E(Y))) \\
&= E(XY) - XE(Y) - YE(X) + E(X)E(Y) \\
&= E(XY) - E(X)E(Y) + E(X)E(Y) \\
&= E(XY) - E(X)E(Y) \\
\end{align*} \]

again notice \( \text{Cov}(X, X) = \text{Var}(X) \)

Also this is the form in our earlier discussion about variance of a sum of random variables.