

More facts about covariance:

If X_1, \dots, X_n are random variables that are not necessarily independent, know $\text{Var}(X_1 + \dots + X_n) = \sum_{i=1}^n \text{Var}(X_i) + 2 \sum_{i < j} \text{Cov}(X_i, X_j)$

If X_i, X_j are independent, then

$$\begin{aligned} \text{Cov}(X_i, X_j) &= E(X_i X_j) - E(X_i)E(X_j) \\ &\quad \downarrow \text{by indep} \\ &= E(X_i)E(X_j) - E(X_i)E(X_j) \\ &= 0 \end{aligned}$$

For this reason, i.e. since " X_i, X_j independent" implies " $\text{Cov}(X_i, X_j) = 0$ " then if X_1, \dots, X_n are all independent, or even just pairwise independent, then $\text{Var}(X_1 + \dots + X_n) = \sum_{i=1}^n \text{Var}(X_i) + 2 \sum_{i < j} \underbrace{\text{Cov}(X_i, X_j)}_{= 0 \text{ by indep}} = \sum_{i=1}^n \text{Var}(X_i)$.