

Conditional Expectation

Idea: Suppose we know the value of some random variable Y , e.g., suppose $Y=y$ is given. Then we want to find the conditional expectation of another random variable, e.g., X .

$E(X|Y=y)$ is the conditional expected value of X , given $Y=y$.

If X is a discrete random variable, then

$$E(X|Y=y) = \sum_x x \underbrace{p_{X|Y}(x|y)}_{\text{conditional probability mass function of } X, \text{ given } Y=y}$$

If X is a continuous random variable, then

$$E(X|Y=y) = \int_{-\infty}^{\infty} x \underbrace{f_{X|Y}(x|y)}_{\text{conditional probability density function of } X, \text{ given } Y=y} dx$$

In both cases, we are taking an expected value over all possible values of X (either adding or integrating, depending on what kind of variable X is),

and we are using either the conditional mass or conditional density of X , given $Y=y$ (a specific value).