

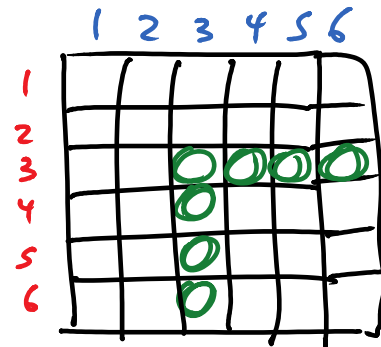
Example Roll two fair six-sided dice.

Let X denote the maximum value.

Let Y denote the minimum value.

Find $E(X | Y=3)$

$$P_{X|Y}(x|3) = \begin{cases} \frac{1}{7} & x=3 \\ \frac{2}{7} & x=4 \\ \frac{2}{7} & x=5 \\ \frac{2}{7} & x=6 \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{aligned} E(X | Y=3) &= (3)P_{X|Y}(3|3) + (4)P_{X|Y}(4|3) + (5)P_{X|Y}(5|3) + (6)P_{X|Y}(6|3) \\ &= \underline{(3)} \left(\underline{\frac{1}{7}} \right) + \underline{(4)} \left(\underline{\frac{2}{7}} \right) + \underline{(5)} \left(\underline{\frac{2}{7}} \right) + \underline{(6)} \left(\underline{\frac{2}{7}} \right) \\ &\quad \left(\text{as always with conditional probability mass functions,} \right. \\ &\quad \left. \text{they must sum to 1: } \frac{1}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = 1 \right) \\ &= \underline{\underline{\frac{33}{7}}} \end{aligned}$$