

Example Suppose  $X$  and  $Y$  have joint density  $f_{X,Y}(x,y) = 40 e^{-3x-5y}$  for  $0 < x < y$   
 $= 0$  otherwise

Find the expected value of  $X$ , given  $Y=y$ .

First, we need  $f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{40 e^{-3x-5y}}{\frac{40}{3}(e^{-5y} - e^{-8y})}$  for  $0 < x < y$   
 $= \int_0^y 40 e^{-3x-5y} dx = \frac{40}{3}(e^{-5y} - e^{-8y})$

$E(X|Y=y) = \int_0^y (X) \frac{40 e^{-3x-5y}}{\frac{40}{3}(e^{-5y} - e^{-8y})} dx = \frac{1}{3} - \frac{y}{e^{2y} - 1}$   
 notice  $y$  is fixed

For instance  
 $E(X|Y=2) = \int_0^2 (X) \frac{40 e^{-3x-5(2)}}{\frac{40}{3}(e^{-5(2)} - e^{-8(2)})} dx = \frac{1}{3} - \frac{2}{e^{2(2)} - 1} = \underline{0.3284}$