

Example (continued) X, Y have joint density $f_{X,Y}(x,y) = 40e^{-3x-5y}$ for $0 < x < y$
= 0 otherwise

Find $E(Y|X=x)$

$$\text{Need } f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{40e^{-3x-5y}}{8e^{-8x}} \text{ for } 0 < x < y$$

$$\int_x^{\infty} 40e^{-3x-5y} dy = 8e^{-8x}$$

$$f_{Y|X}(y|x) = \begin{cases} 5e^{5x-5y} & \text{for } 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$

$$E(Y|X=x) = \int_x^{\infty} (y)(5e^{5x-5y}) dy = \frac{1}{5} + x$$

E.g. if we use $x=2$

$$E(Y|X=2) = \int_2^{\infty} (y)(5e^{5(2)-5y}) dy = \frac{1}{5} + 2 = 2.2.$$