Example Suppose the amount of tuition students owe is independent, and suppose each student owes, on average, $5,000. Suppose we pick a random number of students, with 20 students picked on average. What is the average amount of tuition that the selected students owe, altogether? (Assuming the # of students picked is independent from the amounts they owe.)

Let \( Y \) denote the number of students picked. Once \( Y = y \) is known, let \( X_1, \ldots, X_y \) be the tuition amounts.

\[
E(X_1 + \ldots + X_y) = E\left( E(X_1 + \ldots + X_y \mid Y=y) \right)
\]

\[
= E\left( \frac{5000 + 5000 + \ldots + 5000}{y} \mid Y=y \right)
\]

\[
= E(5000Y) = 5000E(Y) = (5000)(20) = 100,000.
\]

More generally, if the \( X_j \)'s have the same expected values

\[
E(X_1 + \ldots + X_y \mid Y=y) = yE(X_1)
\]

\[
E(X_1 + \ldots + X_y) = YE(X_1)
\]

\[
E\left( E(X_1 + \ldots + X_y \mid Y) \right) = E(YE(X_1)) = E(X_1)E(Y)
\]

If \( Y \) is integer valued:

\[
E(E(X_1 + \ldots + X_y \mid Y)) = \sum_y E(X_1 + \ldots + X_y \mid Y=y)P(Y=y)
\]

\[
= \sum_y yE(X_1)P(Y=y)
\]

\[
= E(X_1)\sum_y yP(Y=y)
\]

\[
= E(X_1)E(Y).
\]