Markov Inequality

Version 1: If $X$ is any random variable (not necessarily positive) and $a$ is any positive constant, then

$$\Pr(|X| \geq a) \leq \frac{E(|X|)}{a}.$$ 

Why? Let $Y$ indicate whether $|X| \geq a$, i.e. $Y=1$ if $|X| \geq a$ and $Y=0$ otherwise.

Idea is that $Y \leq \frac{|X|}{a}$ always.

Two cases to check: If $Y=0$ then $Y=0 \leq \frac{|X|}{a}$.
If $Y=1$ then $a \leq |X|$ so $Y=1 \leq \frac{|X|}{a}$.

Since $Y \leq \frac{|X|}{a}$ always,

$$E(Y) \leq \frac{E(|X|)}{a} = \frac{E(|X|)}{a} \Pr(|X| \geq a).$$

Second version: If $X$ is a random variable that is always $\geq 0$ i.e. nonnegative, then Markov inequality applies to $X$ with no need to take absolute value, i.e. $\Pr(X \geq a) \leq \frac{E(X)}{a}$ for $a>0$. 