Markov Inequality

Version 1 If X is any random variable (not necessarily positive) and "a" is any positive constant, then

$$\boxed{P(|X| \geq a) \leq \frac{E(|X|)}{a}.}$$

Why? Let Y indicate whether $|X| \geq a$, i.e. $Y = 1$ if $|X| \geq a$
$= 0$ otherwise.

Idea is that $Y \leq \frac{|X|}{a}$ always.

Two cases to check: If $Y=0$ then $Y = 0 \leq \frac{|X|}{a}$
If $Y=1$ then $a \leq |X|$ so $Y = 1 \leq \frac{|X|}{a}$.

Since $Y \leq \frac{|X|}{a}$ always, $\underbrace{E(Y)}_{= P(|X| \geq a)} \leq E\left(\frac{|X|}{a}\right) = \frac{E(|X|)}{a}$

Second version: If X is a random variable that is always ≥ 0 i.e. nonnegative, then Markov inequality applies to X with no need to take absolute value, i.e. $P(X \geq a) \leq \frac{E(X)}{a}$ for $a > 0$.