

Examples of the Markov inequality

① Suppose that the average salary of a university professor on a given campus is \$82,000 a year.

Then the probability that a randomly chosen professor's salary exceeds \$95,000 a year is at most .8632.

Why? Let X be the salary of the randomly chosen professor.

$$P(X \geq 95000) \leq \frac{E(X)}{95000} = \frac{82000}{95000} = 0.8632.$$

② Suppose that customers in a given town receive 5.2 pieces of mail on average (in a given day). Find an upper bound on the probability of 8 or more pieces in a given day.

Let X be the amount of mail a randomly chosen person receives.

$$P(X \geq 8) \leq \frac{E(X)}{8} = \frac{5.2}{8} = 0.65$$

So $P(X \geq 8) \leq 0.65$.

Idea in both examples — in fact, in all examples for the Markov inequality — is that a random variable cannot be too far above its mean too often; if it was too far above its mean with a sufficiently large probability, then the mean would need to be larger than it is! This is just an intuitive way of thinking.