Chebyshev's Inequality

$$P(|X-E(X)| \geq k) \leq \frac{Var(X)}{k^2}, \quad \text{for } k>0$$

Why? Use Markov's Inequality with $(X-E(X))^2$ instead of X and k^2 instead of a.

Markov's Inequality says:

$$P(\underbrace{(X-E(X))^2 \geq k^2}_{\text{both positive}}) \leq \frac{E((X-E(X))^2)}{k^2}$$

$$\boxed{P(|X-E(X)| \geq k) \leq \frac{Var(X)}{k^2}}$$

another version, use $k\sigma_X$ instead of k

$k^2 Var(X)$ instead of k^2, we get:

$$P(|X-E(X)| \geq k\sigma_X) \leq \frac{\cancel{Var(X)}}{k^2 \cancel{Var(X)}}$$

$$P(|X-E(X)| \geq k\sigma_X) \leq \frac{1}{k^2}.$$

i.e. The probability that X is more than k standard deviations away from its mean is no larger than $\frac{1}{k^2}$.

Equivalently,

$$P(|X-E(X)| \leq k\sigma_X) \geq 1-\frac{1}{k^2}$$

i.e. the probability X is within k standard deviations of its mean is at least $1-\frac{1}{k^2}$.