

Recreate our earlier example, using these general formulas.

Suppose  $X_1, X_2$  have density  $f_X(x) = 3e^{-3x}$  for  $x > 0$   
 $= 0$  otherwise

$$F_X(x) = 1 - e^{-3x} \text{ for } x > 0 \\ = 0 \text{ otherwise}$$

and suppose  $X_1, X_2$  are independent.

We can use the general formulas we learned to see:

$X_{(1)}, X_{(2)}$  (the first and second order statistics, respectively,  
i.e.  $X_{(1)}$  is the  $\min(X_1, X_2)$ ,  $X_{(2)}$  is the  $\max(X_1, X_2)$ )

has joint density

$$f_{X_{(1)}, X_{(2)}}(x_1, x_2) = 2! 3e^{-3x_1} 3e^{-3x_2} \text{ for } 0 < x_1 < x_2$$

and  $X_{(1)}$  has density

$$f_{X_{(1)}}(x_1) = \binom{2}{0, 1, 1} 3e^{-3x_1} (1 - e^{-3x_1})^0 (e^{-3x_1})^1 \\ = 2 \\ = 6e^{-6x_1} \text{ for } x_1 > 0$$

and  $X_{(2)}$  has density

$$f_{X_{(2)}}(x_2) = \binom{2}{1, 1, 0} 3e^{-3x_2} (1 - e^{-3x_2})^1 (e^{-3x_2})^0 \\ = 2 \\ = 6e^{-3x_2} (1 - e^{-3x_2}) \text{ for } x_2 > 0$$

So the general formulas seem to work for the specific example we calculated earlier.