Transformation of a random variable

Say \( X \) is a continuous Uniform random variable on \([0,5]\).

Define \( Y = \pi X^2 \) i.e. if we treat \( X \) as the radius of a circle, then \( Y \) is the area of the circle.

Two ways to get \( E(Y) \).

1. \( E(Y) = E(\pi X^2) = \int_0^5 (\pi x^2) \left( \frac{1}{5} \right) dx = \frac{25\pi}{3} \)

2. Find CDF of \( Y \), then density of \( Y \), then expected value of \( Y \).

(We learn more about \( Y \) this way.)

\[
F_Y(a) = P(Y \leq a) = P(\pi X^2 \leq a) = P(X \leq \sqrt{\frac{a}{\pi}}) = \frac{\sqrt{\frac{a}{\pi}} - 0}{5 - 0}
\]

for \( 0 < a \leq \pi \cdot 5^2 = 25\pi \)

\[
f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{y} \left( \frac{1}{5\sqrt{\pi}} \right) = \frac{1}{10\sqrt{\pi}y}
\]

\[
E(Y) = \int_0^{25\pi} (y) \frac{1}{10\sqrt{\pi}y} dy = \frac{25\pi}{3} \text{ as before!}
\]