STAT/MA 41600
Practice Problems: August 27, 2014

1. Choosing points at random.

(a.) A point is chosen at random inside in the triangle in Figure 1. What is the sample space? [Please give mathematical expression(s), rather than just (x, y) is "in the triangle".]

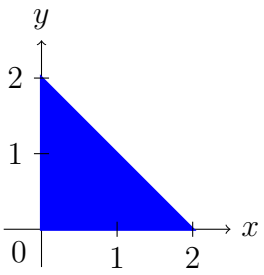

Figure 1: A triangle.

(b.) A point is chosen at random in the quadrilateral in Figure 2. What is the sample space? [Hint: It might be helpful to give bounds on the x coordinate and then give bounds on the y coordinate.]

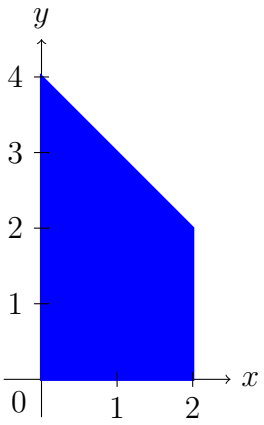

Figure 2: A quadrilateral.

2. Gloves. A matching pair of blue gloves, a matching pair of red gloves, and one lone white right-handed glove are in a drawer. The gloves are pulled out of the drawer, one at a time.

Suppose the color and the "hand" of the gloves are both noted as all five gloves are consecutively pulled out of the drawer. Then, for instance, a possible outcome is:

("blue right", "white right", "red left", "red right", "blue left").

(a.) How many outcomes are in the sample space?

(b.) Now suppose that only the color (not the hand) of the glove is noted as the gloves are removed. This drastically shrinks the size of the sample space. For instance, one possible outcome would be:

("blue", "white", "red", "red", "blue").

Now how many outcomes are in the sample space?

3. Seating arrangements. Alice, Bob, Catherine, Doug, and Edna are randomly assigned seats at a circular table in a perfectly circular room. Assume that rotations of the table do not matter, so there are exactly 24 possible outcomes in the sample space.

Bob and Catherine are married. Doug and Edna are married. When people are married they love to sit beside each other. In how many of these 24 outcomes are both married couples sitting together and therefore happy?

4. Abstract art. A painter has three different jars of paint colors available, namely, green, yellow, and purple. She wants to paint something abstract, so she blindfolds herself, randomly dips her brush, and paints on the canvas. She continues trying paint jars until she finally gets some purple onto the canvas (her assistant will tell her when this happens) and then she stops. Assume that she does not repeat any of the jars because her assistant removes a jar once it has been used. So the sample space is

$$S = \{(P), (G, P), (Y, P), (Y, G, P), (G, Y, P)\}.$$

How many events can be made using this sample space?

5. Sum of three dice. Roll three distinguishable dice (e.g., assume that there is a way to tell them apart, for instance, that the dice are three different colors). There are $6 \times 6 \times 6 = 216$ possible outcomes.

For $3 \leq j \leq 18$, define A_j as the event that the sum of the dice equals j. How many outcomes are in each A_j? (For instance, A_3 contains only one outcome, namely, $(1, 1, 1)$. Similarly, A_{18} contains only one outcome, namely, $(6, 6, 6)$. These are filled in already, in the table below. Your table entries should sum to 216 altogether.)

event	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
number of outcomes	1							

event	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}
number of outcomes								1

Hint: It might be helpful to think about the 11 possible events that are possible with just 2 dice, described here:

x	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$