STAT/MA 41600
Practice Problems: August 29, 2014

1. Choosing a page at random.

A student buys a brand new calculus textbook that has 1000 pages, each numbered with 3 digits, from 000 to 999. She randomly opens the book to a page and starts to read! Assume that any of the 1000 pages are equally likely to be chosen.

Find the probability that the page number she chooses contains at least one "5" as a digit. (For instance, each page in the range 500 to 599 contains a "5", of course, at the start. Also, page 605 has a "5", and page 257 contains a "5", and page 055 contains a "5", etc.) Hint: Use inclusion-exclusion.

2. Gloves. A matching pair of blue gloves, a matching pair of red gloves, and one lone white right-handed glove are in a drawer. The gloves are pulled out of the drawer, one at a time.

(a.) Suppose that a person is looking for the white glove. He repeatedly does the following: He pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he replaces the glove in the drawer and starts to check again, i.e., he reaches blindly into the drawer of 5 gloves. He continues to do this over and over until he finds the white glove, and then he stops. Let A_j denote the event that he successfully discovers the white glove (for the first time) on his jth attempt. Find $P(A_j)$.

(b.) Now suppose that he searches for the white glove but, if he pulls a different colored glove from the drawer, he does not replace it. So he pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he *permanently discards* the glove and starts to check again, i.e., he reaches blindly into the drawer with the gloves that remain. He continues to do this over and over again until he finds the white glove, and then he stops. Let B_j denote the event that he successfully discovers the white glove (for the first time) on his jth attempt. Find $P(B_1), P(B_2), P(B_3), P(B_4)$, and $P(B_5)$.

3. Seating arrangements. Alice, Bob, Catherine, Doug, and Edna are randomly assigned seats at a circular table in a perfectly circular room. Assume that rotations of the table do not matter, so there are exactly 24 possible outcomes in the sample space.

Bob and Catherine are married. Doug and Edna are married. When people are married they love to sit beside each other.

Let A_j denote the event that exactly j of the married couples are happy because they are sitting together. Find $P(A_0)$ and $P(A_1)$ and $P(A_2)$.

4. Abstract art. A painter has three different jars of paint colors available, namely, green, yellow, and purple. She wants to paint something abstract, so she blindfolds herself, randomly dips her brush, and paints on the canvas. She continues trying paint jars until she finally gets some purple onto the canvas (her assistant will tell her when this happens) and then she stops. Assume that she does not repeat any of the jars because her assistant removes a jar once it has been used. So the sample space is

$$S = \{(P), (G, P), (Y, P), (Y, G, P), (G, Y, P)\}.$$

Find the probabilities of each of the following events:

EVENT:	Probability of the event:
$\{(P)\}$	
$\{(G, P), (Y, P)\}$	
$\{(G, P), (G, Y, P)\}$	
$\{(Y, G, P), (G, Y, P)\}$	
$\{(P), (Y, P)\}$	

5. Maximum of three dice. Roll three distinguishable dice (e.g., assume that there is a way to tell them apart, for instance, that the dice are three different colors). There are $6 \times 6 \times 6 = 216$ possible outcomes.

Let B_k be the event that the maximum value that appears on all three dice when they are rolled is less than or equal to k. Find $P(B_1), P(B_2), P(B_3), P(B_4), P(B_5)$, and $P(B_6)$. If you prefer, you are welcome to just give a general formula that covers all six of these cases, i.e., you are welcome to just give a formula for $P(B_k)$ itself.