

1. Choosing a page at random.

A student buys a brand new calculus textbook that has 1000 pages, each numbered with 3 digits, from 000 to 999. She randomly opens the book to a page and starts to read! Assume that any of the 1000 pages are equally likely to be chosen.

Let A be the event that the first digit on the chosen page is a “5” (for example, the page could be 572). Let B be the event that the second and third digits on the chosen pages are both “5” (for example, the page could be 455).

Are A and B independent? Why? Justify your answer.

2. Gloves. A matching pair of blue gloves, a matching pair of red gloves, and one lone white right-handed glove are in a drawer. The gloves are pulled out of the drawer, one at a time.

Suppose that a person is looking for the white glove. He repeatedly does the following: He pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he replaces the glove in the drawer and starts to check again, i.e., he reaches blindly into the drawer of 5 gloves. He continues to do this over and over until he finds the white glove, and then he stops.

Let A denote the event that he pulls out a red glove during this process. In other words, A denotes the event that he finds a red glove before a white glove. Find $P(A)$.

3. Seating arrangements. Alice, Bob, Catherine, Doug, and Edna are randomly assigned seats at a circular table in a perfectly circular room. Assume that rotations of the table do not matter, so there are exactly 24 possible outcomes in the sample space.

Bob and Catherine are married. Doug and Edna are married. When people are married they love to sit beside each other.

Let T denote the event that Bob and Catherine are sitting next to each other. Let U be the event that Alice and Bob are sitting next to each other. Are events T and U independent? Why? Justify your answer.

4. Abstract art. A painter has three different jars of paint colors available, namely, green, yellow, and purple. She wants to paint something abstract, so she blindfolds herself, randomly dips her brush, and paints on the canvas. She continues trying paint jars **until she has used all three of them** (this is different than the story from the last two days), and then she stops. Assume that she does not repeat any of the jars because her assistant removes a jar once it has been used. So the sample space is

$$S = \{(G, P, Y), (G, Y, P), (P, G, Y), (P, Y, G), (Y, G, P), (Y, P, G)\}.$$

Let A be the event that purple is found in the second jar tested by the painter. Let B be the event that green is found before yellow. Are events A and B independent? Why? Justify your answer.

5. Even versus four or less. Roll a die. Let A be the event that the outcome on the die is an even number. Let B be the event that the outcome on the die is 4 or smaller. Let C be the event that the outcome on the die is 3 or larger.

Are A and B independent? Why? Justify your answer.

Are B and C independent? Why? Justify your answer.