STAT/MA 41600
Practice Problems: September 3, 2014
Solutions by Mark Daniel Ward

**1. Choosing a page at random.** Yes, the events are independent. We compute

$$P(A) = 100/1000 = 1/10; \qquad P(B) = 10/1000 = 1/100; \qquad P(A \cap B) = 1/1000.$$

So $P(A)P(B) = P(A \cap B)$. So $A, B$ are independent.

**2. Gloves.** Using the ideas from the notes, about finding a good result (here, a red glove) before a bad one (here, a white glove), we can write $p = 2/5$ and $q = 1/5$, and the desired probability is $\frac{p}{p+q} = \frac{2/5}{2/5+1/5} = 2/3$.

As a another (very intuitive) method of solution, just look at the first draw which is not blue. Such a draw is red with probability 2/3 (as desired), or white with probability 1/3.

If you prefer to grind out the answer, it might be easier to compute the complement, i.e., that the white glove arrives without ever having seen a red glove. For this to happen in exactly $j$ draws has probability:

$$\overbrace{(2/5)(2/5)\cdots(2/5)}^{j-1}(1/5) = (2/5)^{j-1}(1/5).$$

So the probability of getting white before red appears is

$$\sum_{j=1}^{\infty}(2/5)^{j-1}(1/5) = (1/5)\sum_{j=0}^{\infty}(2/5)^j = \frac{1/5}{1-2/5} = 1/3.$$

Thus the probability of getting red before white (i.e., the probability of the complementary event) is 2/3, as we claimed above.

**3. Seating arrangements.** No, events $T$ and $U$ are not independent; rather, $T, U$ are dependent. For any given pair of people, the probability of them sitting next to each other is $2/4 = 1/2$, because the first member of the pair can sit anywhere and the other member of the pair can sit next to her/him in two out of the four possible seats, all of which are equally likely. Thus $P(T) = 1/2$ and $P(U) = 1/2$.

On the other hand, to compute $P(T \cap U)$, we need Bob to sit by Alice and Catherine. Bob can sit anywhere, and then Alice always has 2 out of the 4 remaining possible places to sit beside Bob, and this leaves Catherine 1 out of the 3 remaining possible places to sit beside Bob. Thus $P(T \cap U) = (2/4)(1/3) = 1/6 \neq 1/4 = P(T)P(U)$. So $T, U$ are dependent.

**4. Abstract art.** Yes, $A, B$ are independent. First, $P(A) = 1/3$ because purple is equally likely to be found in any of the three possible trials. Also, $P(B) = 1/2$ because green is equally likely to be found before or after yellow. Finally, $P(A \cap B) = 1/6$ because the 6 outcomes are equally likely, and $A \cap B$ only occurs if $(G, P, Y)$ is the outcome.

**5. Even versus four or less.** Yes, $A, B$ are independent. We have

$$\begin{aligned} P(A) &= P(\{2,4,6\}) = 3/6 = 1/2; \\ P(B) &= P(\{1,2,3,4\}) = 4/6 = 2/3; \\ P(A \cap B) &= P(\{2,4\}) = 2/6 = 1/3 = P(A)P(B). \end{aligned}$$

No, $B, C$ are not independent; these events are dependent. We have

$$\begin{aligned} P(B) &= P(\{1,2,3,4\}) = 4/6 = 2/3; \\ P(C) &= P(\{3,4,5,6\}) = 4/6 = 2/3; \\ P(B \cap C) &= P(\{3,4\}) = 2/6 = 1/3 \neq 4/9 = P(B)P(C). \end{aligned}$$