

STAT/MA 41600
Practice Problems: September 15, 2014

1. Butterflies. Alice, Bob, and Charlotte are looking for butterflies. They look in three separate parts of a field, so that their probabilities of success do not affect each other.

- Alice finds 1 butterfly with probability 17%, and otherwise does not find one.
- Bob finds 1 butterfly with probability 25%, and otherwise does not find one.
- Charlotte finds 1 butterfly with probability 45%, and otherwise does not find one.

Let X be the number of butterflies that they catch altogether. Let Y be the number of people who do not catch a butterfly.

Find the joint mass $p_{X,Y}(x, y)$ of X and Y .

2. Dependence/independence among dice rolls. A student rolls a die until the first “4” appears. Let X be the numbers of rolls required until (and including) this first “4.” After this is completed, he begins rolling again until he gets a “3.” Let Y be the number of rolls, after the first “4”, up to (and including) the next “3.” E.g., if the sequence of rolls is 213662341261613 then $X = 8$ and $Y = 7$. Are X and Y independent? Justify your answer.

3. Wastebasket basketball. Chris tries to throw a ball of paper in the wastebasket behind his back (without looking). He estimates that his chance of success each time, regardless of the outcome of the other attempts, is $1/3$. Let X be the number of attempts required. If he is not successful within the first 5 attempts, then he quits, and he lets $X = 6$ in such a case.

Let Y indicate whether he makes the basket successfully within the first three attempts. Thus $Y = 1$ if his first, second, or third attempt is successful, and $Y = 0$ otherwise.

Find the conditional mass of X given Y . You will need to list 12 values altogether, i.e., you need to compute $p_{X|Y}(x | y)$ for $1 \leq x \leq 6$ and $0 \leq y \leq 1$.

4. Two 4-sided dice. Consider some special 4-sided dice. Roll two of these dice. Let X denote the minimum of the two values that appear, and let Y denote the maximum of the two values that appear.

Find the joint mass $p_{X,Y}(x, y)$ of X and Y .

Find the joint CDF $F_{X,Y}(x, y)$ of X and Y . It suffices to give the values $F_{X,Y}(x, y)$ when x and y are integers between 1 and 4. You do not have to list any other values; only these 16 possibilities will suffice.

5. Pick two cards. Pick two cards at random from a well-shuffled deck of 52 cards (pick them simultaneously, i.e., grab two cards at once—so they are not the same card!). There are 12 cards which are considered face cards (4 Jacks, 4 Queens, 4 Kings). There are 4 cards with the value 10. Let X be the number of face cards in your hand; let Y be the number of 10's in your hand.

Are X and Y dependent or independent? Carefully justify your answer.