STAT/MA 41600

Practice Problems: September 19, 2014

1. Butterflies. Alice, Bob, and Charlotte are looking for butterflies. They look in three separate parts of a field, so that their probabilities of success do not affect each other.

- Alice finds 1 butterfly with probability 17%, and otherwise does not find one.
- Bob finds 1 butterfly with probability 25%, and otherwise does not find one.
- Charlotte finds 1 butterfly with probability 45%, and otherwise does not find one.

Let X be the number of butterflies that they catch altogether.

Write X as the sum of three indicator random variables, X_1, X_2, X_3 that indicate whether Alice, Bob, Charlotte (respectively) found a butterfly. Then $X = X_1 + X_2 + X_3$. Find the expected value of X by finding the expected value of the sum of the indicator random variables.

2. Dependence/independence among dice rolls. A student rolls a die until the first "4" appears. Let X be the numbers of rolls required until (and including) this first "4." After this is completed, he begins rolling again until he gets a "3." Let Y be the number of rolls, after the first "4", up to (and including) the next "3." E.g., if the sequence of rolls is 213662341261613 then $X = 8$ and $Y = 7$.

Let A_j be the event containing all outcomes in which "j or more rolls" are required to get the first "4." Let X_j indicate whether or not A_j occurs. Then $X = X_1 + X_2 + X_3 + \cdots$. Find the expected value of X by finding the expected value of the sum of the indicator random variables.

Let B_j be the event containing all outcomes in which "j or more rolls" are required, after the first "4", until he gets a "3". Let Y_j indicate whether or not B_j occurs. Then $Y = Y_1 + Y_2 + Y_3 + \cdots$. Find the expected value of Y by finding the expected value of the sum of the indicator random variables.

3. Wastebasket basketball. Chris tries to throw a ball of paper in the wastebasket behind his back (without looking). He estimates that his chance of success each time, regardless of the outcome of the other attempts, is $1/3$. Let X be the number of attempts required. If he is not successful within the first 5 attempts, then he quits, and he lets $X = 6$ in such a case.

Let A_j be the event containing all outcomes in which "j or more attempts" are required to get the basket. Let X_j indicate whether or not A_j occurs. Then $X = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$. Find the expected value of X by finding the expected value of the sum of the indicator random variables.

4. Two 4-sided dice. Consider some special 4-sided dice. Roll two of these dice. Let X denote the minimum of the two values that appear, and let Y denote the maximum of the two values that appear.

Let A_j be the event containing all outcomes in which the minimum of the two dice is "j or greater." Let X_j indicate whether or not A_j occurs. Then $X = X_1 + X_2 + X_3 + X_4$. Find the expected value of X by finding the expected value of the sum of the indicator random variables.

5. Pick two cards. Pick two cards at random from a well-shuffled deck of 52 cards (pick them simultaneously, i.e., grab two cards at once—so they are not the same card!). There are 12 cards which are considered face cards (4 Jacks, 4 Queens, 4 Kings). There are 4 cards with the value 10. Let X be the number of face cards in your hand; let Y be the number of 10's in your hand.

Before looking at the cards, put one in your left hand and one in your right hand. Let X_1 and X_2 indicate, respectively, whether the cards in your left and right hands (respectively) are face cards. Then $X = X_1 + X_2$. Find the expected value of X by finding the expected value of the sum of the indicator random variables.

Before looking at the cards, put one in your left hand and one in your right hand. Let Y_1 and Y_2 indicate, respectively, whether the cards in your left and right hands (respectively) are 10's. Then $Y = Y_1 + Y_2$. Find the expected value of Y by finding the expected value of the sum of the indicator random variables.