

STAT/MA 41600
Practice Problems: September 22, 2014

1a. Variance of an Indicator. Suppose event A occurs with probability p , and X is an indicator for A , i.e., $X = 1$ if A occurs, or $X = 0$ otherwise. We already know $\mathbb{E}(X) = p$. Find $\text{Var}(X)$.

1b. Butterflies. Alice, Bob, and Charlotte are looking for butterflies. They look in three separate parts of a field, so that their probabilities of success do not affect each other.

- Alice finds 1 butterfly with probability 17%, and otherwise does not find one.
- Bob finds 1 butterfly with probability 25%, and otherwise does not find one.
- Charlotte finds 1 butterfly with probability 45%, and otherwise does not find one.

Let X be the number of butterflies that they catch altogether. Find the variance of X .

2. Appetizers. At a restaurant that sells appetizers:

- 8% of the appetizers cost \$1 each,
- 20% of the appetizers cost \$2 each,
- 32% of the appetizers cost \$3 each,
- 40% of the appetizers cost \$4 each.

An appetizer is chosen at random, and X is its price. Each appetizer has 7% sales tax. So $Y = 1.07X$ is the amount paid on the bill (in dollars). Find the variance of Y .

3. Gloves. A matching pair of blue gloves, a matching pair of red gloves, and one lone white right-handed glove are in a drawer. The gloves are pulled out of the drawer, one at a time.

Suppose that a person is looking for the white glove. He repeatedly does the following: He pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he *permanently discards* the glove and starts to check again, i.e., he reaches blindly into the drawer with the gloves that remain. He continues to do this over and over again until he finds the white glove, and then he stops. Let X be the number of draws that are necessary to find the white glove for the first time.

Find $\mathbb{E}(X)$.

Find $\mathbb{E}(X^2)$.

Find $\text{Var}(X)$.

Find $\mathbb{E}(X^3)$.

4. Two 4-sided dice. Consider some special 4-sided dice. Roll two of these dice. Let X denote the minimum of the two values that appear, and let Y denote the maximum of the two values that appear.

Find the variance of X .

[Caution: If X_j is an indicator of whether the minimum of the two dice is “ j or greater”—as in the previous homework—then $X = X_1 + X_2 + X_3 + X_4$, but the X_j ’s are dependent. So we cannot just sum the variances. We need to find the mass of X and then compute the expected value and variance by hand.]

5. Pick two cards. Pick two cards at random from a well-shuffled deck of 52 cards (pick them simultaneously, i.e., grab two cards at once—so they are not the same card!). There are 12 cards which are considered face cards (4 Jacks, 4 Queens, 4 Kings). There are 4 cards with the value 10. Let X be the number of face cards in your hand; let Y be the number of 10's in your hand.

Find the variance of X .

Find the variance of Y .

- 6. Design your own problem and solution.** Create your own problem about either:
1. the expected value of a function of a discrete random variable, or
 2. the variance of a discrete random variables.

Design your problem in such a way that you would find it enjoyable and also interesting (i.e., not completely trivial) if you found this problem in a probability book. Please provide the answer for your problem too.