STAT/MA 41600
Practice Problems: September 24, 2014

1. Winnings and Losing. Suppose that a person wins a game of chance with probability 0.40, and loses otherwise. If he wins, he earns 5 dollars, and if he loses, then he loses 4 dollars.

(a.) What is his expected gain or loss?

(b.) What is the variance of his gain or loss?

(c.) Find constants a, b such that if $X = 0$ when he loses and $X = 1$ when he wins, then $Y = aX + b$ is his earnings. [Hint: solve $5 = a(1) + b$ and $-4 = a(0) + b$. Also: X is a Bernoulli.] Verify your results above by finding $\mathbb{E}(Y)$ and $\mathrm{Var}(Y)$ with this method.

2. Telemarketers. One out of every eight calls to your house is a telemarketer. You will record whether the next call is a telemarketer.

(a.) What is the probability that the next time the phone rings, it will be a telemarketer?

(b.) If the phone calls are independent, what is the probability the 3rd call after dinner will be a telemarketer?

(c.) If you lose 30 seconds of your time every time you have to talk to a telemarketer, what is the expected amount of time you will lose on the next phone call?

(d.) What is the standard deviation of time you will lose on the next phone call?

3. Dating. You randomly call some of your friends who could be potential partners for a dance. You think that they all respond to your requests independently of each other, and you estimate that each one is 7% likely to accept your request. Let X_j indicate the event that you need to call j or more people until you find someone who accepted your invitation. So

$$X = X_1 + X_2 + X_3 + \cdots$$

is the total number of people that you need to call until someone accepts your invitation.

Find $\mathbb{E}(X)$.

4. Studying. Let X be the number of nights that you spend studying in a 30-day month. Assume that you study, on a given night, with probability .65, independent of the other nights. Write X as the sum of thirty indicators (i.e., as the sum of 30 Bernoulli random variables).

Find $\mathbb{E}(X)$.

Find $\text{Var}(X)$.

5. Shoes. Anne and Jane have shoes spread throughout the dorm room. Anne has 15 pairs of shoes; twenty percent of her shoe collection consists of sandals. Jane has 40 pairs of shoes; ten percent of her shoe collection consists of sandals.

(a.) A shoe is picked at random from the dorm room belonging to Anne and Jane; what is the probability that it is a sandal?

(b.) If a randomly-chosen shoe is chosen from the room (with all shoes equally likely to be chosen), what is the probability that it belongs to Anne?

(c.) If a randomly-chosen shoe is chosen from the room (with all shoes equally likely to be chosen), and upon examination this shoe is seen to be a sandal, what is the probability that it belongs to Anne?