STAT/MA 41600
Practice Problems: September 24, 2014
Solutions by Mark Daniel Ward

1. Winnings and Losing. (a.) His expected gain/loss is $(.4)(5) + (.6)(-4) = -0.40$.
(b.) The variance of his gain or loss is $(.4)(5)^2 + (.6)(-4)^2 - (-0.40)^2 = 19.44$.

(c.) We have $a = 9$ and $b = -4$, so $Y = 9X - 4$. Thus $\mathbb{E}(Y) = 9\mathbb{E}(X) - 4 = 9(.4) - 4 = -0.40$ and $\operatorname{Var}(Y) = 9^2 \operatorname{Var}(X) = (9^2)(.6)(.4) = 19.44$.

2. Telemarketers. (a.) The probability that the next caller is a telemarketer is 1/8.

(b.) The probability that the 3rd caller is a telemarketer is 1/8.

(c.) Let X indicate whether the next caller is a telemarketer. Then we lose $30X$ seconds on the next phone call, so we expected to lose $\mathbb{E}(30X) = 30\mathbb{E}(X) = (30)(1/8) = 30/8 = 15/4 = 3.75$ seconds on the next phone call.

(d.) Again, let X indicate whether the next caller is a telemarketer. Then we lose $30X$ seconds on the next phone call, so the variance of the time lost is $\operatorname{Var}(30X) = 900 \operatorname{Var}(X) = (900)(1/8)(7/8) = 1575/16 = 98.4375$, and the standard deviation of the time lost is $\sigma_{30X} = \sqrt{98.4375} = 9.9216$.

3. Dating. We have $\mathbb{E}(X_j) = P(X_j = 1)$, which is the probability that the first $j - 1$ attempts are unsuccessful, i.e., $(.93)^{j-1}$. So $\mathbb{E}(X) = \sum_{j=1}^{\infty}(.93)^{j-1} = \frac{1}{1-.93} = 1/.07 = 100/7 = 14.29$.

4. Studying. We have $\mathbb{E}(X) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \cdots + \mathbb{E}(X_{30}) = .65 + .65 + \cdots + .65 = (30)(.65) = 19.50$.

Since the X_j's are independent, then $\operatorname{Var}(X) = \operatorname{Var}(X_1) + \operatorname{Var}(X_2) + \cdots + \operatorname{Var}(X_{30}) = (.65)(.35) + (.65)(.35) + \cdots + (.65)(.35) = (30)(.65)(.35) = 6.825$.

5. Shoes. (a.) Altogether $(.20)(15) + (.10)(40) = 7$ out of the $15 + 40 = 55$ shoes are sandals, so the probability is $7/55 = .1272$.

(b.) Exactly 15 of the 55 shoes belong to Anne, so the probability is $15/55 = 3/11 = .2727$.

(c.) There are exactly 7 sandals; each is equally-likely to be chosen, and 3 of them are Anne's, so the probability is $3/7 = .4286$.