

STAT/MA 41600  
Practice Problems: September 26, 2014

**1. Winnings and Losing.** Suppose that a person wins a game of chance with probability 0.40, and loses otherwise. If he wins, he earns 5 dollars, and if he loses, then he loses 4 dollars. He plays the game until he wins for the first time, and then he stops. Assume that the games are independent of each other. Let  $X$  denote the number of games that he must play until (and including) his first win.

(a.) How many games does he expect to play until (and including) his first win?

(b.) What is the variance of the number of games he plays until (and including) his first win?

(c.) What is the probability that he plays 4 or more games altogether?

**2. Winnings and Losing (continued).** Continue to use the scenario from the previous problem. As before, let  $X$  denote the number of games that he must play until (and including) his first win.

(a.) Find a formula for his gain or loss, in terms of  $X$ . I.e., if  $Y$  denotes his gain or loss in dollars, write  $Y$  in terms of  $X$ .

(b.) What is his expected gain or loss (altogether) during the  $X$  games? I.e., what is  $\mathbb{E}(Y)$ ?

(c.) What is the variance of his gain or loss (altogether) during the  $X$  games? I.e., what is  $\text{Var}(Y)$ ?

**3. Telemarketers.** One out of every eight calls to your house is a telemarketer. Assume that the likelihood of telemarketers is independent from call to call. Let  $X$  denote the number of callers until (and including) the next call by a telemarketer.

If  $n$  is a non-negative integer, what is  $P(X > n)$ ?

**4. Dating.** You randomly call friends who could be potential partners for a dance. You think that they all respond to your requests independently of each other, and you estimate that each one is 7% likely to accept your request. Let  $X$  denote the number of phone calls that you make to successfully get a date.

(a.) Find the expected number of people you need to call, i.e.,  $\mathbb{E}(X)$ .

(b.) Find the variance of the number of people you need to call, i.e.,  $\text{Var}(X)$ .

(c.) Given that the first 3 people do not accept your invitation, let  $Y$  denote the additional number of people you need to call ( $Y$  does not include those first 3 people). I.e., Suppose  $X > 3$  is given; then let  $Y = X - 3$ . Under these conditions, what is the mass of  $Y$ ?

**5. Hearts.** You draw cards, one at a time, *with replacement* (i.e., placing them randomly back into the deck after they are drawn), from a shuffled, standard deck of 52 playing cards. Let  $X$  be the number of cards that are drawn to get the first heart that appears.

(a.) How many cards do you expect to draw, to see the first heart?

(b.) Now suppose that you draw five cards (again, with replacement), and none of them are hearts. How many additional cards (not including the first five) do you expect to draw to see the first heart?