STAT/MA 41600
Practice Problems: September 29, 2014
Solutions by Mark Daniel Ward

1. Quidditch Training.
(a.) Since X is Negative Binomial with $r = 4$ successes and with $p = 0.15$ on each trial, then the mass of X is $p_X(x) = \binom{x-1}{3}(.85)^{x-4}(.15)^4$, for $x = 4, 5, 6, \ldots$, and $p_X(x) = 0$ otherwise.

(b.) The probability is $p_X(12) = \binom{11}{3}(.85)^8(.15)^4, = 0.02276$.

(c.) The expected value is $\mathbb{E}(X) = r/p = 4/0.15 = 26.67$.

2. Horcruxes.
(a.) Since X is Negative Binomial with $r = 7$ and $p = 1/3$, then $\mathbb{E}(X) = r/p = 7/(1/3) = 21$.

(b.) As before, using Negative Binomial parameters, $\operatorname{Var} X = qr/p^2 = (2/3)7/(1/3)^2 = 42$.

(c.) The mass of X is $p_X(x) = \binom{x-1}{6}(2/3)^{x-7}(1/3)^7$, for $x = 7, 8, 9, \ldots$, and $p_X(x) = 0$ otherwise. So $p_X(9) = \binom{8}{6}(2/3)^2(1/3)^7 = 112/19683 = 0.00569$.

3. Mandrakes.

(a.) The expected number is $3/0.02 = 150$.

(b.) The variance is $(.98)(3)/(0.02)^2 = 7350$.

4. Divination.

(a.) Lavender earns $Y = (100)(5) - (15)(X - 5) = 575 - 15X$.

(b.) Her expected earnings are $\mathbb{E}(Y) = \mathbb{E}(575 - 15X) = 575 - 15\mathbb{E}(X) = 575 - 15(5/0.12) = -50$.

(c.) Her earnings have variance $\operatorname{Var}(Y) = \operatorname{Var}(575 - 15X) = 15^2 \operatorname{Var}(X) = 15^2 \frac{(0.88)(5)}{(.12)^2} = 68750$.

5. Spells.

(a.) The expected value of X is $\mathbb{E}(X) = 5/.30 + 3/.30 + 20/.30 = 93.33$. This can also be checked by noticing that X is a Negative Binomial random variable with $r = 28$ and $p = .30$, so $\mathbb{E}(X) = r/p = 28/.3 = 93.33$.

(b.) We have $\operatorname{Var}(X) = (.7)(5)/(.30)^2 + (.7)(3)/(.30)^2 + (.7)(20)/(.30)^2 = 217.78$. Also: X is a Negative Binomial with $r = 28$ and $p = .30$, so $\operatorname{Var}(X) = qr/p^2 = (.7)(28)/(.3)^2 = 217.78$.