STAT/MA 41600
Practice Problems: October 1, 2014
Solutions by Mark Daniel Ward

1. Hungry customers.

a. The number X of arrivals in the next 10 minutes is Poisson with average of 5 people. So the probability of exactly 3 arrivals is $P(X = 3) = e^{-5}5^3/3! = .1404$.

b. Again, the number X of arrivals in the next 10 minutes is Poisson with average of 5 people. So the probability of no arrivals is $P(X = 0) = e^{-5}5^0/0! = .0067$.

c. Again, the number X of arrivals in the next 10 minutes is Poisson with average of 5 people. So the probability of at least 3 arrivals is $P(X \geq 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) = 1 - e^{-5}5^0/0! - e^{-5}5^1/1! - e^{-5}5^2/2! = .8753$.

2. Errors in Dr. Ward's book.

a. The number X of errors in the next 100 pages is Poisson with average $(100)(.04) = 4$.

b. Since the number X of errors in the next 100 pages is Poisson with average $(100)(.04) = 4$, then the probability of exactly 5 errors is $P(X = 5) = e^{-4}4^5/5! = .1563$.

3. Telemarketers.

a. The number X of telemarketers per day is Poisson with average $\lambda = 3/7 = .4285$ per day. So the mass is $P(X = j) = e^{-3/7}(3/7)^j/j!$ for $j = 0, 1, 2, 3, \ldots$, and $P(X = j) = 0$ otherwise.

b. Since the number X of telemarketers per day is Poisson with average $\lambda = 3/7 = .4285$ per day, then the probability that none of them call your house on a given day is $P(X = 0) = e^{-\lambda}\lambda^0/0! = e^{-\lambda} = .6514$.

c. Since the number X of telemarketers per day is Poisson with average $\lambda = 3/7 = .4285$ per day, then the probability that exactly 2 of them call your house on a given day is $P(X = 2) = e^{-\lambda}\lambda^2/2! = .0598$.

4. Superfans.

a. Altogether we expert 14 such fans per hour, so we expect $(3)(14) = 42$ of them during the next 3 hours.

b. Since the number X of such fans has average 14 per hour, then the average during the next 20 minutes is $(14)(20/60) = 4.6667$. So the probability of exactly one such fan is $P(X = 1) = e^{-4.6667}(4.6667)^1/1! = 0.04388$.

5. Shoppers.

a. The number X of men in a 10-second period is Poisson with average $(12)(10/60) = 2$, so the probability of 1 man in the next ten seconds is $P(X = 1) = e^{-2}2^1/1! = 2e^{-2}$. The number Y of women in a 10-second period is Poisson with average $(15)(10/60) = 5/2$, so the probability of 2 women in the next ten seconds is $P(Y = 2) = e^{-5/2}(5/2)^2/2! = \frac{25}{8}e^{-5/2}$. So the desired probability, since X and Y are independent, is $(2e^{-2})(\frac{25}{8}e^{-5/2}) = \frac{25}{4}e^{-9/2} = 0.0694$.

b. The number of people per minute is Poisson with average 27 per minute. So the number Z of people in a 5-minute period is Poisson with mean $\mathbb{E}(Z) = (5)(27) = 135$. The expected value and variance of a Poisson is always the same, so $\mathrm{Var}(Z) = 135$ too.