

STAT/MA 41600  
Practice Problems: October 31, 2014  
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1.

a. The probability is  $\int_{60}^{\infty} \frac{1}{30} e^{-x/30} dx = -e^{-x/30} \Big|_{x=60}^{\infty} = e^{-2} = .1353$ .

b. Since the average is  $1/\lambda = 30$  minutes, then the variance is  $1/\lambda^2 = 900$ , so the standard deviation is  $\sqrt{1/\lambda^2} = 30$  minutes.

2. Let  $X$  be the time (in minutes) until the next dessert; let  $Y$  be the time (in minutes) until the next appetizer. The probability is  $P(X < Y) = \int_0^{\infty} \int_x^{\infty} \frac{1}{3} e^{-x/3} \frac{1}{2} e^{-y/2} dy dx = \int_0^{\infty} -\frac{1}{3} e^{-x/3} e^{-y/2} \Big|_{y=x}^{\infty} dx = \int_0^{\infty} \frac{1}{3} e^{-5x/6} dx = -\frac{1/3}{5/6} e^{-5x/6} \Big|_{x=0}^{\infty} = 2/5$ .

3. Using the CDF of  $X$ , we have  $P(X \leq 20) = F_X(20) = 1 - e^{-20(1/20)} = 1 - e^{-1}$ . Similarly  $P(Y \leq 20) = 1 - e^{-1}$  and  $P(Z \leq 20) = 1 - e^{-1}$ . Since the  $X, Y, Z$  are independent, then  $P(\max(X, Y, Z)) = P(X \leq 20)P(Y \leq 20)P(Z \leq 20) = (1 - e^{-1})^3 = 0.2526$ .

4. The company expects to pay  $\int_0^3 (0) \frac{1}{1.5} e^{-x/1.5} dx + \int_3^{\infty} (72)(100)(x - 3) \frac{1}{1.5} e^{-x/1.5} dx = (72)(100) \int_0^{\infty} x \frac{1}{1.5} e^{-(x+3)/1.5} dx = (72)(100) e^{-2} \int_0^{\infty} x \frac{1}{1.5} e^{-x/1.5} dx$ . Notice  $\int_0^{\infty} x \frac{1}{1.5} e^{-x/1.5} dx$  is the expected value of  $X$ , i.e., is 1.5. So the company expects to pay  $(72)(100)(e^{-2})(1.5) = 1461.62$  dollars.

5. The probability is

$\int_0^{10} \int_x^{\infty} (\frac{1}{10})(\frac{1}{5}) e^{-y/5} dy dx = \int_0^{10} -\frac{1}{10} e^{-y/5} \Big|_{y=x}^{\infty} dx = \int_0^{10} \frac{1}{10} e^{-x/5} dx = \frac{1}{2} \int_0^{10} \frac{1}{5} e^{-x/5} dx$ , but the last integral is just the CDF of an exponential with average of 5, evaluated at 10. So the overall probability is  $\frac{1}{2}(1 - e^{-10/5}) = \frac{1}{2}(1 - e^{-2}) = 0.4323$ .