STAT/MA 41600
Practice Problems: November 10, 2014
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1. a. We compute $P(X \leq 10) = P\big(\frac{X-4.2}{\sqrt{50.41}} \leq \frac{10-4.2}{\sqrt{50.41}}\big) = P(Z \leq 0.82) = 0.7939$.

b. We compute $P(X \leq 0) = P\big(\frac{X-4.2}{\sqrt{50.41}} \leq \frac{0-4.2}{\sqrt{50.41}}\big) = P(Z \leq -0.59) = P(0.59 \leq Z) = 1 - P(Z \leq 0.59) = 1 - 0.7224 = 0.2776$.

c. Combining the work above, we have $P(0 \leq X \leq 10) = P(X \leq 10) - P(X \leq 0) = 0.7939 - 0.2776 = 0.5163$.

2. We compute $P(70 \leq X) = P\big(\frac{70-72.5}{6.9} \leq \frac{X-72.5}{6.9}\big) = P(-0.36 \leq Z) = P(Z \leq 0.36) = 0.6406$.

3. We compute $0.3898 = P(a \leq Z \leq .54) = P(Z \leq .54) - P(Z \leq a) = .7054 - P(Z \leq a)$. Thus $P(Z \leq a) = .7054 - 0.3898 = 0.3156$. [Note, in particular, that now we can see a will be negative.] Equivalently, we have $P(-a \leq Z) = 0.3156$, so $P(Z \leq -a) = 1 - 0.3156 = .6844$. So from the normal chart, we have $-a = 0.48$, so $a = -0.48$.

4. a. We compute $P(66 \leq X) = P\big(\frac{66-64}{12.8} \leq \frac{X-64}{12.8}\big) = P(0.16 \leq Z) = 1 - P(Z \leq 0.16) = 1 - 0.5636 = 0.4364$.

b. Let $X_1, \ldots, X_{10}$ be indicator random variables corresponding to the first, ..., tenth person, so that $X_j = 1$ if the jth person has height 66 inches or taller, or $X_j = 0$ otherwise. Then $\mathbb{E}(X_1 + \cdots + X_{10}) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_{10}) = 0.4364 + \cdots + 0.4364 = 4.364$.

5. *Method #1:* We compute $0.1492 = P(X \leq x) = P\big(\frac{X-22}{\sqrt{8}} \leq \frac{x-22}{\sqrt{8}}\big) = P\big(Z \leq \frac{x-22}{\sqrt{8}}\big)$. Taking complements on both sides yields $1 - 0.1492 = 1 - P\big(Z \leq \frac{x-22}{\sqrt{8}}\big) = P\big(\frac{x-22}{\sqrt{8}} \leq Z\big)$. Simplifying (and switching directions on the right-hand-side) yields $0.8508 = P\big(Z \leq -\frac{x-22}{\sqrt{8}}\big)$. So $-\frac{x-22}{\sqrt{8}} = 1.04$, and thus $x = (\sqrt{8})(-1.04) + 22 = 19.06$.

Method #2: We start with $0.1492 = P(Z \leq z)$, which is not on the table, so taking complements gives $1 - 0.1492 = 1 - P(Z \leq z) = P(z \leq Z)$, so $0.8508 = P(Z \leq -z)$. Thus $-z = 1.04$, so $z = -1.04$. Now that we have the value of z we need, we can return to the original statement, to get: $0.1492 = P(Z \leq -1.04) = P(\mu_X + \sigma_X Z \leq \mu_X + \sigma_X(-1.04)) = P(X \leq 22 - (\sqrt{8})(1.04)) = P(X \leq 19.06)$. So the desired quantity is $x = 19.06$.