STAT/MA 41600
Practice Problems: November 12, 2014

1a. If X_1, X_2, X_3, X_4, X_5 are independent normal random variables that each have average 8.2 and variance 32.49, then the sum of the X_j's is normal. Thus, if we divide the sum of the X_j's by 5, we get the average of the X_j's, which is normal too: $Y = \frac{X_1+X_2+X_3+X_4+X_5}{5}$.

Find the expected value $\mathbb{E}(Y)$, and find the variance $\text{Var}(Y)$.

1b. If $X_1, X_2, \ldots, X_n$ are independent normal random variables that each have average μ and variance σ^2, then the sum of the X_j's is normal. Thus, if we divide the sum of the X_j's by n, we get the average of the X_j's, which is normal too: $Y = \frac{X_1+X_2+\cdots+X_n}{n}$.

Find the expected value $\mathbb{E}(Y)$, and find the variance $\text{Var}(Y)$.

2. Assume that the quantity of money in a randomly-selected checking account is normal with mean \$1325 and standard deviation \$25. Also assume that the amounts in different accounts of different people are independent. Let X be the sum of the money contained (altogether) in three randomly-chosen people's accounts. Find the probability that X exceeds \$4000.

3. The time that it takes a random person to get a haircut is normally distributed, with an average of 23.8 minutes and a standard deviation of 5 minutes. Assume that different people have independent times of getting their hair cut. Find the probability that, if there are four customers in a row (with no gaps in between), they will all be finished getting their hair cut in 1.5 hours (altogether) or less.

4. Assume that the height of an American female is normal with expected value $\mu = 64$ inches and standard deviation $\sigma = 12.8$ inches. Also assume that different women have independent heights.

Measure the heights $X_1, \ldots, X_{10}$ of ten women. Let $Y = \frac{X_1 + X_2 + \cdots + X_{10}}{10}$ denote their average height. Find the probability that Y exceeds 60 inches.

5. The quantity of sugar X (measured in grams) in a randomly-selected piece of candy is normally distributed, with expected value $\mathbb{E}(X) = \mu = 22$ and variance $\text{Var}(X) = \sigma^2 = 8$. Assume that different pieces of candy have independent quantities of sugar. Find the probability that, in a handful containing 7 pieces of candy, there are 150 or more grams of sugar.