

STAT/MA 41600  
Practice Problems: November 14, 2014  
Solutions by Mark Daniel Ward

1. Let  $X_1, \dots, X_{30}$  denote the 30 waiting times. Then  $\mathbb{E}(X_j) = 1/2$  and  $\text{Var} X_j = 1/4$ , so  $\mathbb{E}(X_1 + \dots + X_{30}) = (30)(1/2) = 15$  and  $\text{Var}(X_1 + \dots + X_{30}) = (30)(1/4) = 7.5$ . So  $P(X_1 + \dots + X_{30} > 14) = P(\frac{X_1 + \dots + X_{30} - 15}{\sqrt{7.5}} > \frac{14 - 15}{\sqrt{7.5}}) \approx P(Z > -.37) = P(Z < .37) = .6443$ .

2. Let  $X_1, \dots, X_{30}$  be indicator random variables that denote whether the 30 students are happy with their items, i.e.,  $X_j = 1$  if the  $j$ th student is happy, or  $X_j = 0$  otherwise. Then  $\mathbb{E}(X_j) = .60$  and  $\text{Var} X_j = (.60)(.40) = .24$ , so  $\mathbb{E}(X_1 + \dots + X_{30}) = (30)(.60) = 18$  and  $\text{Var}(X_1 + \dots + X_{30}) = (30)(.24) = 7.2$ . So, using continuity correction since the  $X_j$ 's are integer-valued random variables,  $P(X_1 + \dots + X_{30} \geq 20) = P(X_1 + \dots + X_{30} \geq 19.5) = P(\frac{X_1 + \dots + X_{30} - 18}{\sqrt{7.2}} \geq \frac{19.5 - 18}{\sqrt{7.2}}) \approx P(Z \geq .56) = 1 - P(Z < .56) = 1 - .7123 = .2877$ .

3. a. We compute  $\mathbb{E}(X) = \int_0^{10} x \frac{(10-x)^3}{2500} dx = \int_0^{10} (10-u) \frac{u^3}{2500} du = 2$ , and  $\mathbb{E}(X^2) = \int_0^{10} x^2 \frac{(10-x)^3}{2500} dx = \int_0^{10} (10-u)^2 \frac{u^3}{2500} du = 20/3$ , so  $\text{Var}(X) = 20/3 - 2^2 = 8/3$ .

b. Let  $X_1, \dots, X_{200}$  be the delays of the 200 people. So  $\mathbb{E}(X_1 + \dots + X_{200}) = (200)(2) = 400$  and  $\text{Var}(X_1 + \dots + X_{200}) = (200)(8/3) = 1600/3$ . So  $P(X_1 + \dots + X_{200} > 420) = P(\frac{X_1 + \dots + X_{200} - 400}{\sqrt{1600/3}} > \frac{420 - 400}{\sqrt{1600/3}}) \approx P(Z > .87) = 1 - P(Z \leq .87) = 1 - .8078 = .1922$ .

4. Let  $X_1, \dots, X_{100}$  be the completion times of the 100 people. So  $\mathbb{E}(X_1 + \dots + X_{100}) = (100)(3.5) = 350$  and  $\text{Var}(X_1 + \dots + X_{100}) = (100)(1/4) = 25$ . So  $P(348 < X_1 + \dots + X_{100} < 352) = P(\frac{348 - 350}{\sqrt{25}} < \frac{X_1 + \dots + X_{100} - 350}{\sqrt{25}} < \frac{352 - 350}{\sqrt{25}}) \approx P(-.4 < Z < .4)$ . We break this up as  $P(Z < .4) - P(Z \leq -.4) = P(Z < .4) - P(Z \geq .4) = P(Z < .4) - (1 - P(Z < .4)) = 2P(Z < .4) - 1 = (2)(.6554) - 1 = .3108$ .

5. We have  $\mathbb{E}(X_1 + \dots + X_{12}) = (12)(0.99) = 11.88$  and  $\text{Var}(X_1 + \dots + X_{12}) = (12)(.03^2) = .0108$ . So  $P(Y > 1) = P(\frac{X_1 + \dots + X_{12}}{12} > 1) = P(X_1 + \dots + X_{12} > 12) = P(\frac{X_1 + \dots + X_{12} - 11.88}{\sqrt{.0108}} > \frac{12 - 11.88}{\sqrt{.0108}}) \approx P(Z > 1.15) = 1 - P(Z \leq 1.15) = 1 - .8749 = .1251$ .