STAT/MA 41600
Practice Problems #2: November 14, 2014
Solutions by Mark Daniel Ward

1. Let X be the number of flights that are on time. Then X is Binomial with $n = 2000$ and $p = 0.70$, so $P(X > 1420) = P(X > 1420.5) = P\left(\frac{X-(2000)(0.70)}{\sqrt{(2000)(0.70)(0.30)}} > \frac{1420.5-(2000)(0.70)}{\sqrt{(2000)(0.70)(0.30)}}\right) \approx P(Z > 1.00) = 1 - P(Z \leq 1.00) = 1 - .8413 = 0.1587$.

2. Let X be the number of students who attend.

Then X is a Binomial random variable with $n = 400$, $p = 0.60$, so $P(230 \leq X \leq 250) = P(229.5 \leq X \leq 250.5) = P\left(\frac{229.5-(400)(0.60)}{\sqrt{(400)(0.60)(0.40)}} \leq \frac{X-(400)(0.60)}{\sqrt{(400)(0.60)(0.40)}} \leq \frac{250.5-(400)(0.60)}{\sqrt{(400)(0.60)(0.40)}}\right)$. This is approximately $P(-1.07 \leq Z \leq 1.07) = P(Z \leq 1.07) - P(Z < -1.07) = P(Z \leq 1.07) - P(Z > 1.07) = P(Z \leq 1.07) - (1 - P(Z \leq 1.07)) = 2P(Z \leq 1.07) - 1 = 2(.8577) - 1 = .7154$.

3. Let X be the number of broken crayons.

Then X is a Binomial random variable, $n = 12{,}000$, $p = 0.05$, so $P(580 \leq X \leq 620) = P(579.5 \leq X \leq 620.5) = P\left(\frac{579.5-(12{,}000)(0.05)}{\sqrt{(12{,}000)(0.05)(0.95)}} \leq \frac{X-(12{,}000)(0.05)}{\sqrt{(12{,}000)(0.05)(0.95)}} \leq \frac{620.5-(12{,}000)(0.05)}{\sqrt{(12{,}000)(0.05)(0.95)}}\right)$. This is roughly $P(-0.86 \leq Z \leq 0.86) = P(Z \leq 0.86) - P(Z < -0.86) = P(Z \leq 0.86) - P(Z > 0.86) = P(Z \leq 0.86) - (1 - P(Z \leq 0.86)) = 2P(Z \leq 0.86) - 1 = 2(.8051) - 1 = .6102$.

4. Let X be the number of passengers with the extra screening.

Then X is a Binomial random variable with $n = (8)(180) = 1440$ and $p = 0.06$, so $P(X \geq 80) = P(X \geq 79.5) = P\left(\frac{X-(1440)(0.06)}{\sqrt{(1440)(0.06)(0.94)}} \geq \frac{79.5-(1440)(0.06)}{\sqrt{(1440)(0.06)(0.94)}}\right) \approx P(Z \geq -0.77) = P(Z \leq 0.77) = .7794$.

5. Let X be the number of field goals Jeff makes successfully. Let Y be the number of field goals Steve makes successfully. So we want $P(X > Y)$, i.e., $P(X - Y > 0)$. We see that

$$X - Y = X_1 + X_2 + \cdots + X_{120} - Y_1 - Y_2 - \cdots - Y_{164},$$

where X_j indicates whether Jeff's jth attempt was a success, and Y_j indicates whether Steve's jth attempt was a success. So $X - Y$ is the sum of a large number of independent random variables, and thus $X - Y$ is approximately normal.

We have $\mathbb{E}(X - Y) = (120)(.80) - (164)(.60) = -2.40$, and $\operatorname{Var} X - Y = \operatorname{Var} X + \operatorname{Var} Y = (120)(.80)(.20) + (164)(.60)(.40) = 58.56$. Thus $P(X > Y) = P(X - Y > 0) = P(X - Y > 0.5) = P\left(\frac{X-Y-(-2.40)}{\sqrt{58.56}} > \frac{0.5-(-2.40)}{\sqrt{58.56}}\right) \approx P(Z > 0.38) = 1 - P(Z \leq 0.38) = 1 - .6480 = .3520$.