

STAT/MA 41600  
Practice Problems: December 1, 2014

1. Let  $X$  and  $Y$  have a joint uniform distribution on the triangle with corners at  $(0, 2)$ ,  $(2, 0)$ , and the origin. Find  $\mathbb{E}(Y \mid X = 1/2)$ .

2. Roll two 6-sided dice. Let  $X$  denote the minimum value that appears, and let  $Y$  denote the maximum value that appears.

a. Find  $\mathbb{E}(Y \mid X = 3)$ .

b. Find  $\mathbb{E}(X + Y \mid X = 3)$ . [Hint: Using (a)'s answer, you can solve (b) in one line!]

**3.** Let  $X_1$  and  $X_2$  be independent exponential random variables, each with mean 1. Let  $Y = X_1 + X_2$ . Find  $\mathbb{E}(X_1 | Y = 3)$ .

4. Consider a tray with 8 lemonades and 3 raspberry juices. Alice and Bob each take 1 drink from the tray, without replacement. Assume that all of their choices are equally likely. Let  $X_1$  and  $X_2$  indicate (respectively) if Alice and Bob (respectively) get lemonade. In other words,  $X_1 = 1$  if Alice gets lemonade, or  $X_1 = 0$  otherwise; and  $X_2 = 1$  if Bob gets lemonade, or  $X_2 = 0$  otherwise.

a. Find  $\mathbb{E}(X_1 \mid X_2 = 1)$ .

b. Find  $\mathbb{E}(X_1 \mid X_2 = 0)$ .

5. Sally and David each pick 10 flowers from the case without paying attention to what type of flowers they are picking. There are a large quantity of flowers available, 20% of which are roses. Let  $X$  be the number of roses that Sally picks, and let  $Y$  be the number of roses that the couple picks altogether. Find the number of roses that we expect Sally to pick if the total number of roses picked is  $Y = 12$ .