STAT/MA 41600
Practice Problems: December 3, 2014
Solutions by Mark Daniel Ward

1a. Let X be the studying time. Then $P(X \geq 7) \leq \mathbb{E}(X)/7 = 5/7$.

1b. We have $P(3 \leq X \leq 7) = P(|X - 5| \leq 2)$, but $2 = (8/5)(5/4)$, so $P(3 \leq X \leq 7) = P(|X - 5| \leq (8/5)(5/4)) \geq \frac{(8/5)^2 - 1}{(8/5)^2} = 39/64$.

2. Let X the time between two consecutive sneezes. Then $\mathbb{E}(X) = 35$ and $\sigma_X = 1.5$. So $P(30 \leq X \leq 40) = P(|X - 35| \leq 5)$, but $5 = (10/3)(3/2)$, so $P(30 \leq X \leq 40) = P(|X - 35| \leq (10/3)(3/2)) \geq \frac{(10/3)^2 - 1}{(10/3)^2} = 91/100$.

3. a. Let X be the amount of food eaten. Then $P(X \geq 1000) \leq \mathbb{E}(X)/1000 = 750/1000 = 3/4$.

b. We have $P(X > 1000 \text{ or } X < 500) = P(|X - 750| \geq 250)$, but $250 = (250/100)(100)$, so $P(X > 1000 \text{ or } X < 500) = P(|X - 750| \geq (250/100)(100)) \leq \frac{1}{(250/100)^2} = 4/25$.

4. a. Let X be the number of people needed to find the 25th person who likes artichokes. Then X is Negative Binomial with $r = 25$ and $p = .11$. So $\mathbb{E}(X) = 25/(.11) = 2500/11 = 227.27$.

b. Since X is Negative Binomial with $r = 25$ and $p = .11$ and $q = 1 - p = .89$, then $\text{Var}(X) = qr/p^2 = 222500/121 = 1838.84$.

5. a. The random variable Y is a Gamma random variable with $r = 2$ and $\lambda = 1/10$.

b. We have $\mathbb{E}(Y) = r/\lambda = (2)(10) = 20$.

c. We have $\text{Var}\, Y = r/\lambda^2 = (2)(10^2) = 200$.

d. The density of Y is $f_Y(y) = \frac{(1/10)^2}{\Gamma(2)} y^{2-1} e^{-y/10} = \frac{ye^{-y/10}}{100}$ for $y > 10$, and $f_Y(y) = 0$ otherwise. So $P(Y > 12) = \int_{12}^{\infty} f_Y(y)\, dy = \frac{11}{5} e^{-6/5} = .6626$.