1. Jim cuts wood planks of length $X$ for customers, where $X$ is uniformly distributed between 10 and 14 feet. The price of a piece of wood is $2 per foot, plus a flat-rate surcharge of $2 for Jim’s services. So $Y = 2X + 2$ is the amount he charges for a piece of wood.

a. Find the density $f_Y(y)$. Be sure to specify the interval where $f_Y(y)$ is nonzero.

b. Using the density $f_Y(y)$ of $Y$, find the probability that $Y$ exceeds $28$.

c. Check your answer: Using the density $f_X(x)$ of $X$, find the probability that $Y = 2X + 2$ exceeds $28$. 
2. Let $X$ be the price of a CD during a “lightning sale” on Cyber Monday. The total purchase price (in dollars) is $Y = 1.07X + 3.99$, since there is 7% tax and $3.99$ shipping. Suppose that $X$ is uniform on the interval $[4, 9]$.

a. Find the density $f_Y(y)$. Be sure to specify the interval where $f_Y(y)$ is nonzero.

b. Find the expected purchase price (with tax and shipping) by $E(Y) = \int y f_Y(y) \, dy$.

c. Check your answer: Find the expected purchase price (with tax and shipping) by $E(1.07X + 3.99) = \int_4^9 (1.07x + 3.99) f_X(x) \, dx$. 

3. Let $X$ be a random variable that is uniform on $[3, 6]$.

a. If we make a box with area $Y = (X - 1)(X + 1)$, what is the CDF of $Y$? [Hint: Note that $(2)(4) \leq Y \leq (5)(7)$, i.e., $8 \leq Y \leq 35$.]

b. What is the density $f_Y(y)$ of the area $Y$ of the box?

c. Use the density of $Y$ to get the expected area by integrating, i.e., $E(Y) = \int_{8}^{35} y f_Y(y) \, dy$.

d. Check your answer: Integrate with respect $X$ to get the same expected area, i.e., $E((X - 1)(X + 1)) = \int_{3}^{6} (x - 1)(x + 1)f_X(x) \, dx$. 

4. Let $X$ and $Y$ have a joint uniform distribution on the triangle with corners at $(0, 2)$, $(2, 0)$, and the origin. Find the covariance of $X$ and $Y$. 
5. There are 20 chairs in a circle, and 10 pairs of married individuals. Assume all seating arrangements are equally likely. Let $X$ be the number of couples sitting together, i.e., let $X$ be the number of men who are sitting next to their own wives. Find the variance of $X$.

Hint: Write $X = X_1 + \cdots + X_{10}$, where $X_j$ indicates whether the $j$th couple sits together. Then

$$
\text{Var}(X) = \text{Var}(X_1 + \cdots + X_{10}) = \sum_{j=1}^{10} \text{Var}(X_j) + 2 \sum_{1 \leq i < j \leq 10} \text{Cov}(X_i, X_j).
$$

[There are 10 terms of the first type and 90 terms of the second type.]