

STAT/MA 41600
Practice Problems: December 10, 2014

1. Jim cuts wood planks of length X for customers, where X is uniformly distributed between 10 and 14 feet. The price of a piece of wood is \$2 per foot, plus a flat-rate surcharge of \$2 for Jim's services. So $Y = 2X + 2$ is the amount he charges for a piece of wood.

a. Find the density $f_Y(y)$. Be sure to specify the interval where $f_Y(y)$ is nonzero.

b. Using the density $f_Y(y)$ of Y , find the probability that Y exceeds \$28.

c. Check your answer: Using the density $f_X(x)$ of X , find the probability that $Y = 2X + 2$ exceeds \$28.

2. Let X be the price of a CD during a “lightning sale” on Cyber Monday. The total purchase price (in dollars) is $Y = 1.07X + 3.99$, since there is 7% tax and \$3.99 shipping. Suppose that X is uniform on the interval $[4, 9]$.

a. Find the density $f_Y(y)$. Be sure to specify the interval where $f_Y(y)$ is nonzero.

b. Find the expected purchase price (with tax and shipping) by $\mathbb{E}(Y) = \int y f_Y(y) dy$.

c. Check your answer: Find the expected purchase price (with tax and shipping) by $\mathbb{E}(1.07X + 3.99) = \int_4^9 (1.07x + 3.99) f_X(x) dx$.

3. Let X be a random variable that is uniform on $[3, 6]$.

a. If we make a box with area $Y = (X - 1)(X + 1)$, what is the CDF of Y ? [Hint: Note that $(2)(4) \leq Y \leq (5)(7)$, i.e., $8 \leq Y \leq 35$.]

b. What is the density $f_Y(y)$ of the area Y of the box?

c. Use the density of Y to get the expected area by integrating, i.e., $\mathbb{E}(Y) = \int_8^{35} y f_Y(y) dy$.

d. Check your answer: Integrate with respect X to get the same expected area, i.e., $\mathbb{E}((X - 1)(X + 1)) = \int_3^6 (x - 1)(x + 1) f_X(x) dx$.

4. Let X and Y have a joint uniform distribution on the triangle with corners at $(0, 2)$, $(2, 0)$, and the origin. Find the covariance of X and Y .

5. There are 20 chairs in a circle, and 10 pairs of married individuals. Assume all seating arrangements are equally likely. Let X be the number of couples sitting together, i.e., let X be the number of men who are sitting next to their own wives. Find the variance of X .

Hint: Write $X = X_1 + \cdots + X_{10}$, where X_j indicates whether the j th couple sits together. Then

$$\text{Var}(X) = \text{Var}(X_1 + \cdots + X_{10}) = \sum_{j=1}^{10} \text{Var}(X_j) + 2 \sum_{1 \leq i < j \leq 10} \text{Cov}(X_i, X_j).$$

[There are 10 terms of the first type and 90 terms of the second type.]