STAT/MA 41600
In-Class Problem Set #1: August 26, 2015
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Problem Set 1 Answers

1a. There are 9 bears to choose from each time, so the number of possible outcomes is $9 \times 9 \times 9 \times 9 \times 9 = 9^5 = 59049$.
1b. There are 9 bears for the first choice, 8 bears remaining for the second choice, 7 bears remaining for the third choice, etc., so $9 \times 8 \times 7 \times 6 \times 5 = 15120$ possible outcomes.

An alternative view is this: There are $\binom{9}{5} = \frac{9!}{5!4!} = 126$ ways to select 5 out of the 9 bears, without regard to order, and then $5! = 120$ ways to order them, so there are $(126)(120) = 15120$ ways altogether, if you take order into account.
1c. Similar to (1a), there are 6 bears to choose from each time, so the number of possible outcomes is $6 \times 6 \times 6 \times 6 \times 6 = 6^5 = 7776$.
1d. Similar to (1b), there are $(6)(5)(4)(3)(2) = 720$ possible outcomes. Or, using the alternative view, there are $\binom{6}{5} = \frac{6!}{5!1!} = 6$ ways to select 5 out of the 6 bears, without regard to order, and then $5! = 120$ ways to order them, so there are $(6)(120) = 720$ outcomes.
2ab. There are 4 outcomes, and thus, there are $2^4 = 16$ possible events.
2cd. There are $(4)(3)/2 = 6$ outcomes for the pair of people who go to the store, or equivalently, $\binom{4}{2} = \frac{4!}{2!2!} = 6$ outcomes. So there are $2^6 = 64$ possible events.
3a. Each outcome is a list of 10 coins, so there are $2^{10} = 1024$ possible outcomes.
3b. Since there are 1024 outcomes, there are 2^{1024} possible events.
3c. There are $(9)(8)/2 = 36$ ways to pick which two out of the first nine flips will be heads. This is also $\binom{9}{2} = \frac{9!}{2!7!} = 36$. So there are 36 possible outcomes.
4. In all of the parts of this problem, we multiply and divide by $(1 - a)$, which doesn't change things at all (it is like multiplying by 1), and then we cancel the redundant terms. (I included the $0 \leq a < 1$ condition, to make sure that we have convergence if there are infinitely many terms.)
4a. $\sum_{j=0}^{\infty} a^j = \frac{(\sum_{j=0}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(1+a+a^2+\cdots)+(-a-a^2-a^3-\cdots)}{1-a} = 1/(1-a)$
4b. $\sum_{j=1}^{\infty} a^j = \frac{(\sum_{j=1}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(a+a^2+a^3+\cdots)+(-a^2-a^3-a^4-\cdots)}{1-a} = a/(1-a)$ (Or we can just recognize that (4b) is just "a" times the answer to (4a).)
4c. $\sum_{j=5}^{\infty} a^j = \frac{(\sum_{j=5}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(a^5+a^6+a^7+\cdots)+(-a^6-a^7-a^8-\cdots)}{1-a} = a^5/(1-a)$ (Or we can just recognize that (4c) is just "a^5" times the answer to (4a).)
4d. $\sum_{j=k}^{\infty} a^j = \frac{(\sum_{j=k}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(a^k+a^{k+1}+a^{k+2}+\cdots)+(-a^{k+1}-a^{k+2}-a^{k+3}-\cdots)}{1-a} = a^k/(1-a)$ (Or we can just recognize that (4d) is just "a^k" times the answer to (4a).)
4e. $\sum_{j=5}^{20} a^j = \frac{(\sum_{j=5}^{20} a^j)(1-a)}{(1-a)} = \frac{(a^5+a^6+\cdots+a^{20})+(-a^6-a^7-\cdots-a^{21})}{1-a} = (a^5 - a^{21})/(1-a)$
4f. $\sum_{j=k}^{\ell} a^j = \frac{(\sum_{j=k}^{\ell} a^j)(1-a)}{(1-a)} = \frac{(a^k+a^{k+1}+\cdots+a^{\ell})+(-a^{k+1}-a^{k+2}-\cdots-a^{\ell+1})}{1-a} = (a^k - a^{\ell+1})/(1-a)$