

**Problem Set 1 Answers**

**1a.** There are 9 bears to choose from each time, so the number of possible outcomes is  $9 \times 9 \times 9 \times 9 \times 9 = 9^5 = 59049$ .

**1b.** There are 9 bears for the first choice, 8 bears remaining for the second choice, 7 bears remaining for the third choice, etc., so  $9 \times 8 \times 7 \times 6 \times 5 = 15120$  possible outcomes.

An alternative view is this: There are  $\binom{9}{5} = \frac{9!}{5!4!} = 126$  ways to select 5 out of the 9 bears, without regard to order, and then  $5! = 120$  ways to order them, so there are  $(126)(120) = 15120$  ways altogether, if you take order into account.

**1c.** Similar to (1a), there are 6 bears to choose from each time, so the number of possible outcomes is  $6 \times 6 \times 6 \times 6 \times 6 = 6^5 = 7776$ .

**1d.** Similar to (1b), there are  $(6)(5)(4)(3)(2) = 720$  possible outcomes. Or, using the alternative view, there are  $\binom{6}{5} = \frac{6!}{5!1!} = 6$  ways to select 5 out of the 6 bears, without regard to order, and then  $5! = 120$  ways to order them, so there are  $(6)(120) = 720$  outcomes.

**2ab.** There are 4 outcomes, and thus, there are  $2^4 = 16$  possible events.

**2cd.** There are  $(4)(3)/2 = 6$  outcomes for the pair of people who go to the store, or equivalently,  $\binom{4}{2} = \frac{4!}{2!2!} = 6$  outcomes. So there are  $2^6 = 64$  possible events.

**3a.** Each outcome is a list of 10 coins, so there are  $2^{10} = 1024$  possible outcomes.

**3b.** Since there are 1024 outcomes, there are  $2^{1024}$  possible events.

**3c.** There are  $(9)(8)/2 = 36$  ways to pick which two out of the first nine flips will be heads. This is also  $\binom{9}{2} = \frac{9!}{2!7!} = 36$ . So there are 36 possible outcomes.

**4.** In all of the parts of this problem, we multiply and divide by  $(1 - a)$ , which doesn't change things at all (it is like multiplying by 1), and then we cancel the redundant terms. (I included the  $0 \leq a < 1$  condition, to make sure that we have convergence if there are infinitely many terms.)

**4a.** 
$$\sum_{j=0}^{\infty} a^j = \frac{(\sum_{j=0}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(1+a+a^2+\dots)+(-a-a^2-a^3-\dots)}{1-a} = 1/(1-a)$$

**4b.** 
$$\sum_{j=1}^{\infty} a^j = \frac{(\sum_{j=1}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(a+a^2+a^3+\dots)+(-a^2-a^3-a^4-\dots)}{1-a} = a/(1-a)$$
 (Or we can just recognize that (4b) is just "a" times the answer to (4a).)

**4c.** 
$$\sum_{j=5}^{\infty} a^j = \frac{(\sum_{j=5}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(a^5+a^6+a^7+\dots)+(-a^6-a^7-a^8-\dots)}{1-a} = a^5/(1-a)$$
 (Or we can just recognize that (4c) is just "a<sup>5</sup>" times the answer to (4a).)

**4d.** 
$$\sum_{j=k}^{\infty} a^j = \frac{(\sum_{j=k}^{\infty} a^j)(1-a)}{(1-a)} = \frac{(a^k+a^{k+1}+a^{k+2}+\dots)+(-a^{k+1}-a^{k+2}-a^{k+3}-\dots)}{1-a} = a^k/(1-a)$$
 (Or we can just recognize that (4d) is just "a<sup>k</sup>" times the answer to (4a).)

**4e.** 
$$\sum_{j=5}^{20} a^j = \frac{(\sum_{j=5}^{20} a^j)(1-a)}{(1-a)} = \frac{(a^5+a^6+\dots+a^{20})+(-a^6-a^7-\dots-a^{21})}{1-a} = (a^5 - a^{21})/(1-a)$$

**4f.** 
$$\sum_{j=k}^{\ell} a^j = \frac{(\sum_{j=k}^{\ell} a^j)(1-a)}{(1-a)} = \frac{(a^k+a^{k+1}+\dots+a^{\ell})+(-a^{k+1}-a^{k+2}-\dots-a^{\ell+1})}{1-a} = (a^k - a^{\ell+1})/(1-a)$$